Learning Error-Correcting Representations For Multi-class Problems

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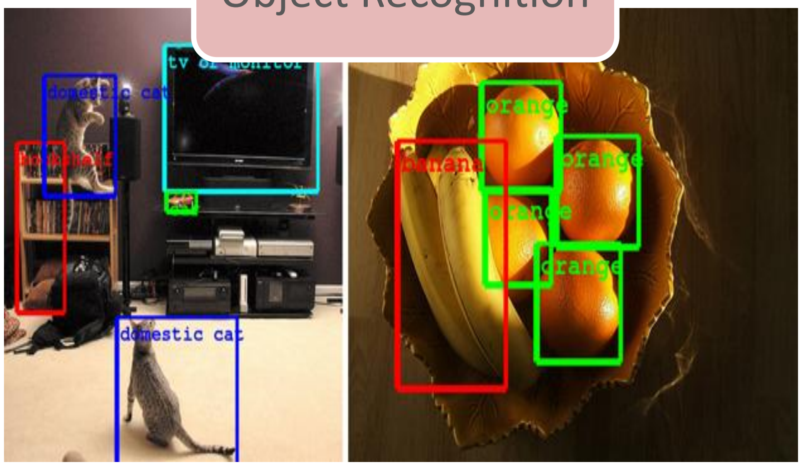
- The **quintessential goal of Artificial Intelligence** is to build machines that are capable of intelligent behavior, by **perceiving**, interacting and learning from their environment.
- Perceptual related tasks share at its core a decision making process.
- Given some sensorial stimulus and previous experience, choose a single option amongst a defined set of possible decisions.
- Most perception tasks can be interpreted as a **classification-categorization** problems.

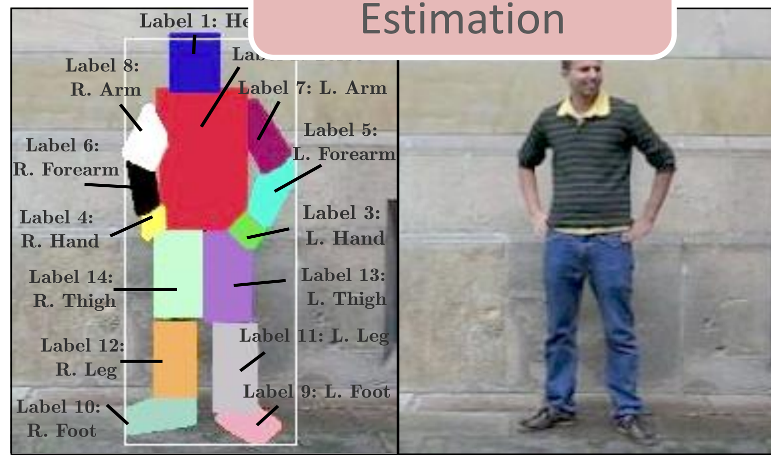


- Classification plays a central role in Computer Vision systems that teach computers how to make sense of images and videos.

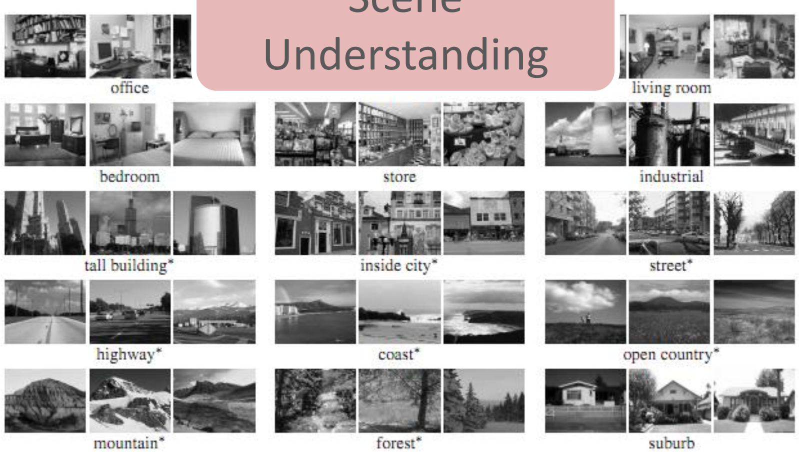
Object Recognition



Human Pose Estimation

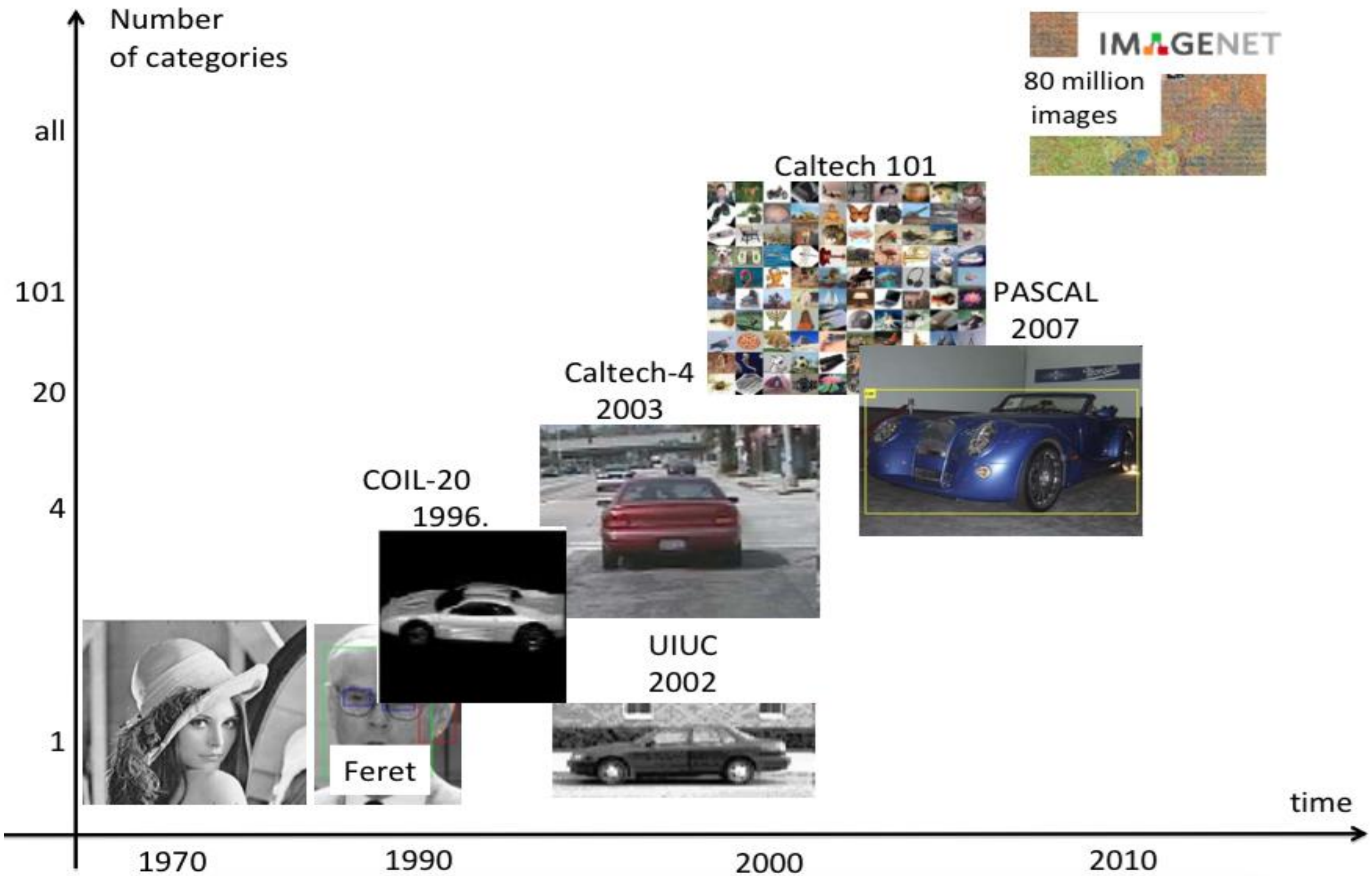


Scene Understanding



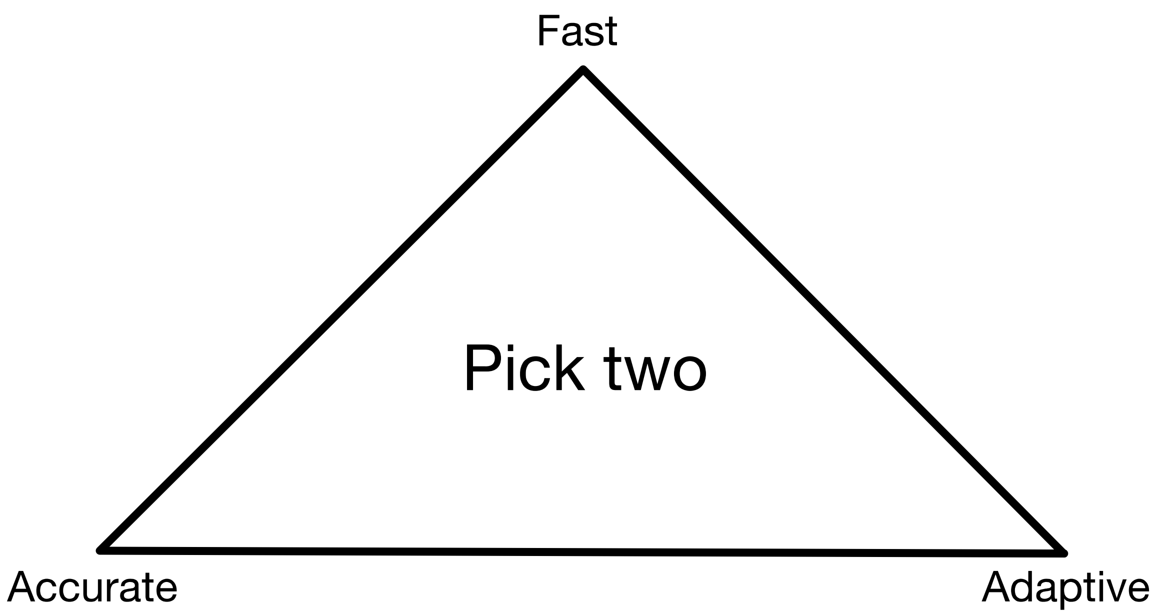
Action Recognition

- The phenomenon of data explosion in image/video databases is clearly perceivable.



The Promise and Perils of Benchmark Datasets and Challenges. David Forsyth, Alyosha Efros, Fei-Fei Li, Antonio Torralba and Andrew Zisserman. Frontiers in Computer Vision Workshop, CVPR 2011.

- This data explosion phenomenon calls for **new developments** in multi-class learning systems.
- The holy grail of classification is a classifier system that is (regardless of the scale of the problem) [1]:
  - **Accurate**
  - **Fast**
  - **Adaptive to data distribution**



[1] Polikar, R. (2006). Ensemble based systems in decision making. Circuits and Systems Magazine, IEEE, 6(3), 21-45.

PROBLEM

- Classification in the traditional **supervised setting**:
  - Learning + Prediction

$$\{\mathbf{x}_i, y_i\} : i \in \{1, \ldots, n\}$$
$$\mathbf{x}_i \in \mathbb{R}^f$$
$$y_i \in \{1, \ldots, k\}$$

  - With $k \leq 2$ we have binary (or one-class) classification:
    - SVMs w/o kernels (object recognition).
    - Adaboost w/o cascading (face recognition).
    - Random Forests (pose estimation).
    - Nearest Neighbors (semantic hashing).
    - Neural Networks (back to end CNNs).
  - While some algorithms naturally extend to $k \geq 2$ (i.e. RF or NN) others cannot be directly applied in the multi-class case.

PROBLEM

- Standard multi-class extensions of binary classifiers share the same spirit:
  - Train one classifier per class.
  - Choosing the classifier with highest score as the prediction.
- What if classifier misses its prediction?
- Should classifiers be trained on groups of classes?
  - **Pros:**
    - Balancing data on classifiers.
    - Leverage the loss in performance of noisy categories.
    - Recover from errors in classifiers!
  - **Cons**:
    - How to obtain the final prediction?

Error-Correcting Output Codes (ECOC)

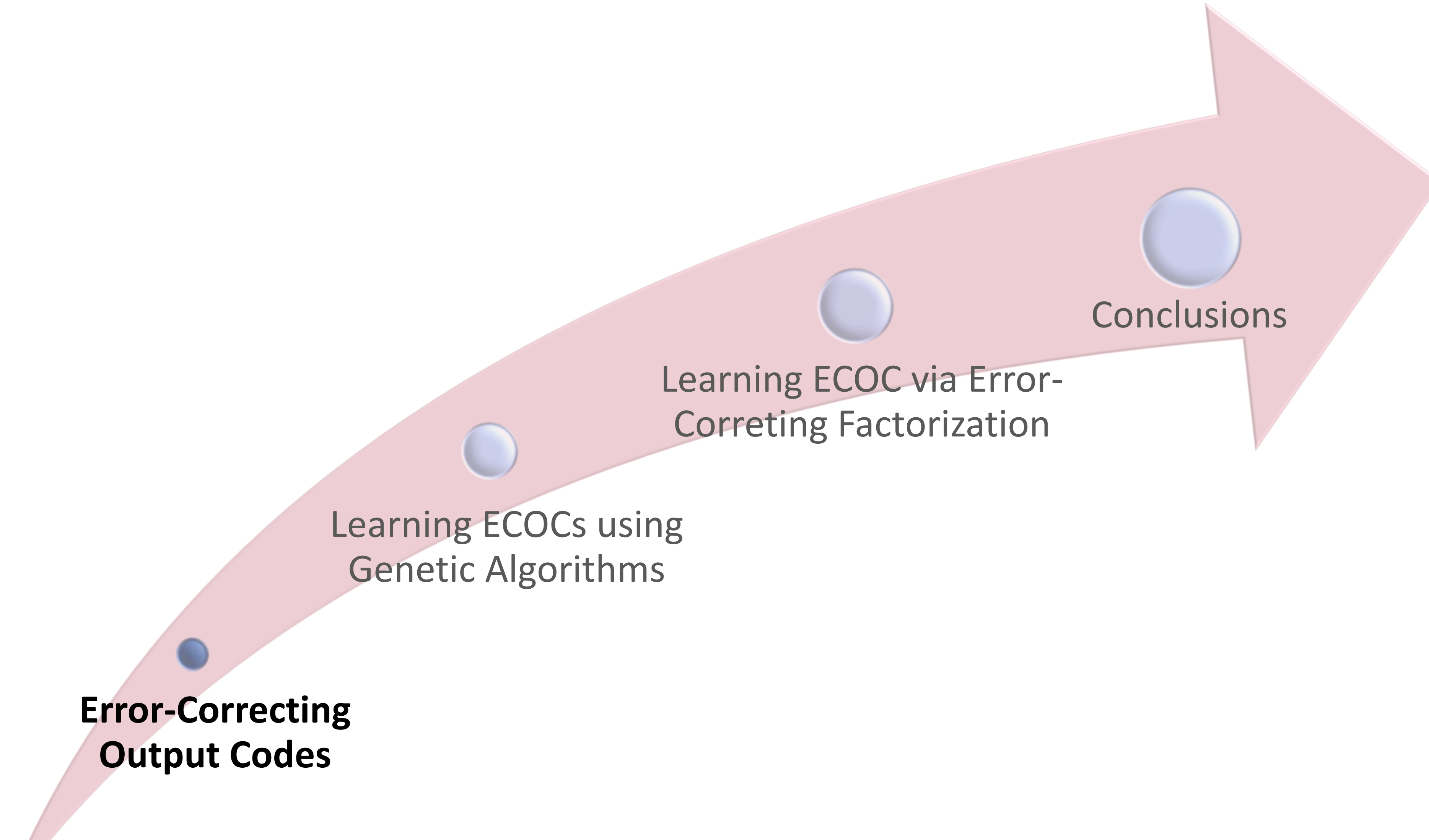
OBJECTIVES

- Develop multi-class classifiers that are accurate, fast and adaptive, within the framework of the Error-Correcting Output Codes:
  - **Accurate**: by using powerful binary classifiers.
  - **Fast**: minimizing the number of classifiers used.
  - **Adaptive**: exploiting multi-class data distribution.

- Deepen into open questions which call for further **study of Error-Correction capabilities** of ECOCs.

- Evaluate our approaches in several Multi-class classification tasks:
  - Localization sites of proteins, Japanese vowel sounds, written letters, etc.
  - Face Recognition, Traffic sign recognition, symbol recognition, etc.

1. **Error-Correcting Output Codes.**
    1. ECOC Introduction.
    2. ECOC Coding.
    3. ECOC Decoding.
    4. ECOC Properties.
2. **Learning ECOCs using Genetic Algorithms.**
    1. Minimal Error-Correcting Output codes.
    2. On the design of an ECOC-compliant Genetic Algorithm.
3. **Learning ECOCs via Error-Correcting Factorization.**
    1. Error-Correcting Capabilities.
    2. Error-Correcting Factorization.
4. **Conclusions.**



Error-Correcting Output Codes (ECOC)

- The **ECOC framework** is a powerful tool to tackle multi-class classification problems.
  - Based on **Error-Correcting principles** of Communications Theory [1].
  - Generalizes standard multi-class decompositions [2].
  - Reduces **both bias and variance** errors [3].

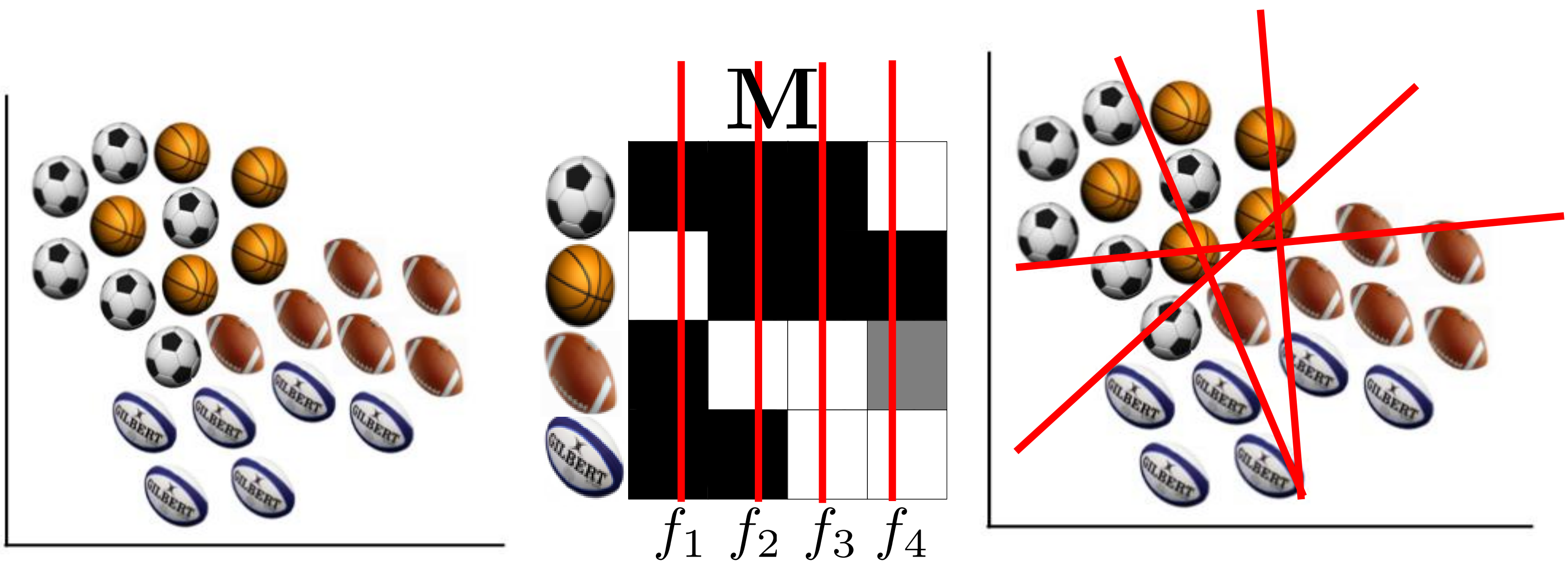
- This framework is composed of two different steps:
  - **Coding**: Decompose a given multiclass problem into a set of binary problems.
  - **Decoding:** Given a test sample, use a decoding measure to determine the prediction.

[1] Dietterich, T. G., & Bakiri, G. (1995). Solving multiclass learning problems via error-correcting output codes. Journal of artificial intelligence research, 263-286.
[2] Rifkin, R., & Klautau, A. (2004). In defense of one-vs-all classification. The Journal of Machine Learning Research, 5, 101-141.
[3] Kong, E. B., & Dietterich, T. G. (1995, July). Error-Correcting Output Coding Corrects Bias and Variance. In ICML (pp. 313-321).

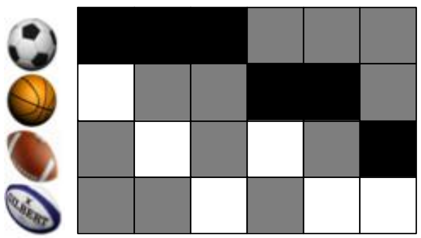
- At the **coding step** a decomposition of the $k-$class problem into l binary problems is computed:

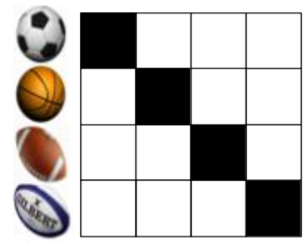
$$\mathbf{M}^{k \times l} \in \{-1, 0, +1\}$$

- **Rows** of the matrix represent the **codewords** of the classes.
- **Columns** of the matrix represent the **binary problems** to be learnt.

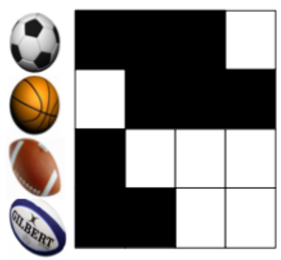


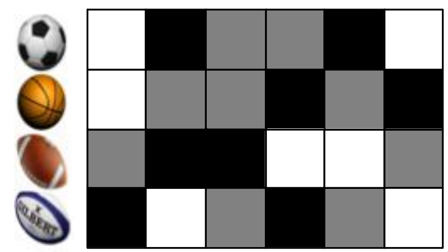
- One vs. One and One vs. All.





- Random (Dense and Sparse).

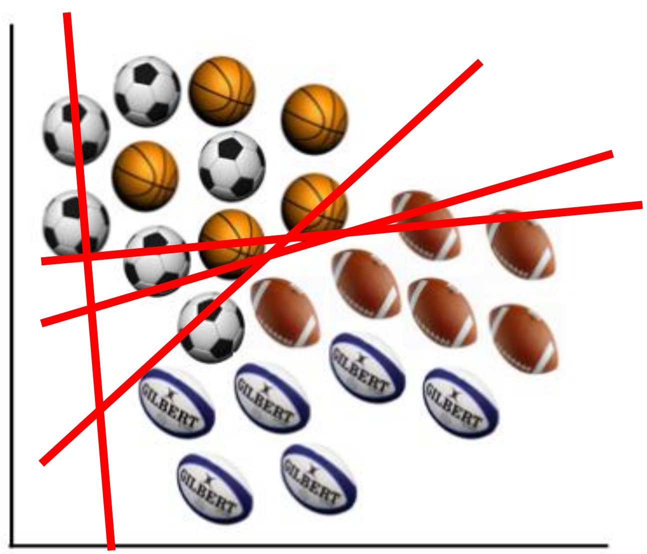


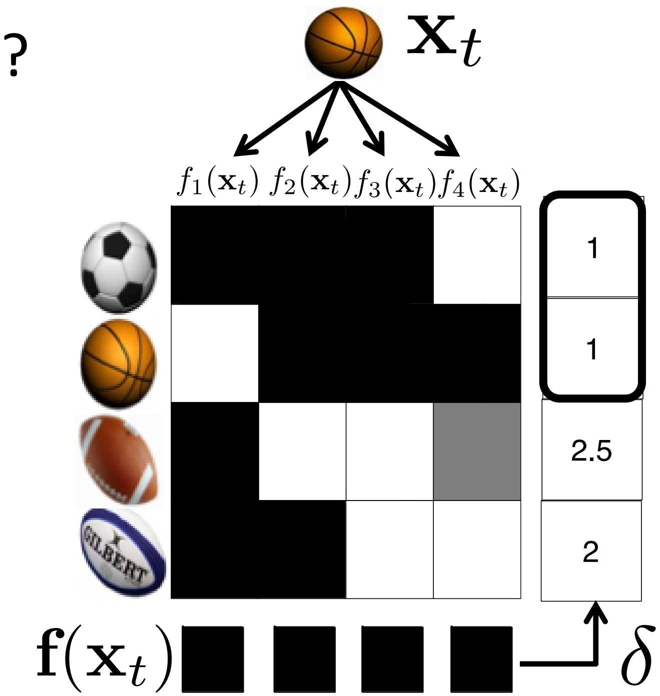


- Problem dependent (Discriminative ECOC [1], Spectral ECOC [2], Genetic Algorithms [3]).

[1] Pujol, O., Radeva, P., & Vitria, J. (2006). Discriminant ecoc: A heuristic method for application dependent design of error correcting output codes. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 28(6), 1007-1012.
[2] Zhang, X., Liang, L., & Shum, H. Y. (2009, September). Spectral error correcting output codes for efficient multiclass recognition. In Computer Vision, 2009 IEEE 12th International Conference on (pp. 1111-1118). IEEE.

- How to perform testing (decoding)?



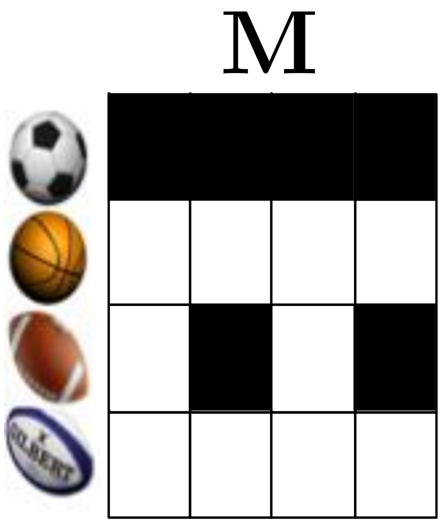


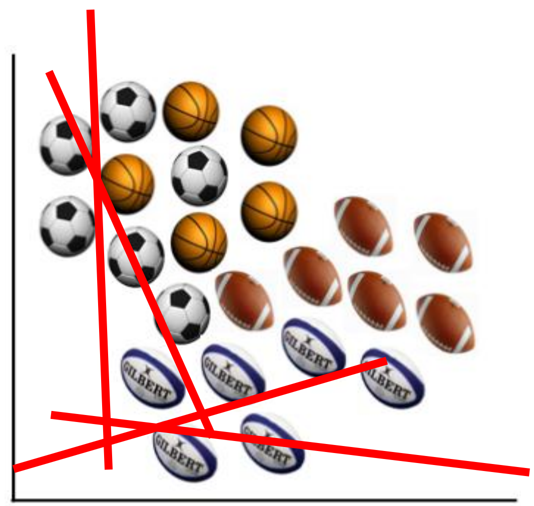
$$\arg\min_i \delta(\mathbf{m}^i, \mathbf{f}(\mathbf{x}_t))$$

- Choices for δ are unlimited [1]:
  - Hamming, Euclidean, Loss-based, probabilistic, etc.
- Even though a classifier missed its prediction, we were able to recover the correct prediction.
  - **Error-Correction!**

[1] Escalera, S., Pujol, O., & Radeva, P. (2010). On the decoding process in ternary error-correcting output codes. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 32(1), 120-134.

- There are some good practices to be followed when building the ECOC coding matrix [1].
  - M must **univocally define all the classes** in the problem (i.e. all the rows of the ECOC matrix must be different).
  - The **binary problems should be uncorrelated** in order to take profit from Error-Correcting principles.
  - **Powerful** (well tuned) **binary classifiers** should be used in order to obtain good classification accuracy.





  - M should **maximize the minimum distance between rows** to profit from Error-Correcting capabilities.

- Formalize the constraints of an ECOC coding matrix [1]:

1. The distance between any pair of rows should be greater or equal than 1.

$$\min(\delta_{AHD}(\mathbf{m}^i, \mathbf{m}^j)) \geq 1 \; \forall_{i,j} \in \{1, \ldots, k\}, i \neq j$$

2. The distance between any pair of columns should be greater or equal than 1.

$$\min(\delta_{HD}(\mathbf{m}_i, \mathbf{m}_j)) \geq 1 \; \forall_{i,j} \in \{1, \ldots, l\}, i \neq j$$

1. A column and its negation are equivalent.

$$\min(\delta_{HD}(\mathbf{m}_i, -\mathbf{m}_j)) \geq 1 \; \forall_{i,j} \in \{1, \ldots, l\}, i \neq j$$

[1] Bautista, M. Á., Escalera, S., Baró, X., & Pujol, O. (2014). On the design of an ECOC-Compliant Genetic Algorithm. Pattern Recognition, 47(2), 865-884.

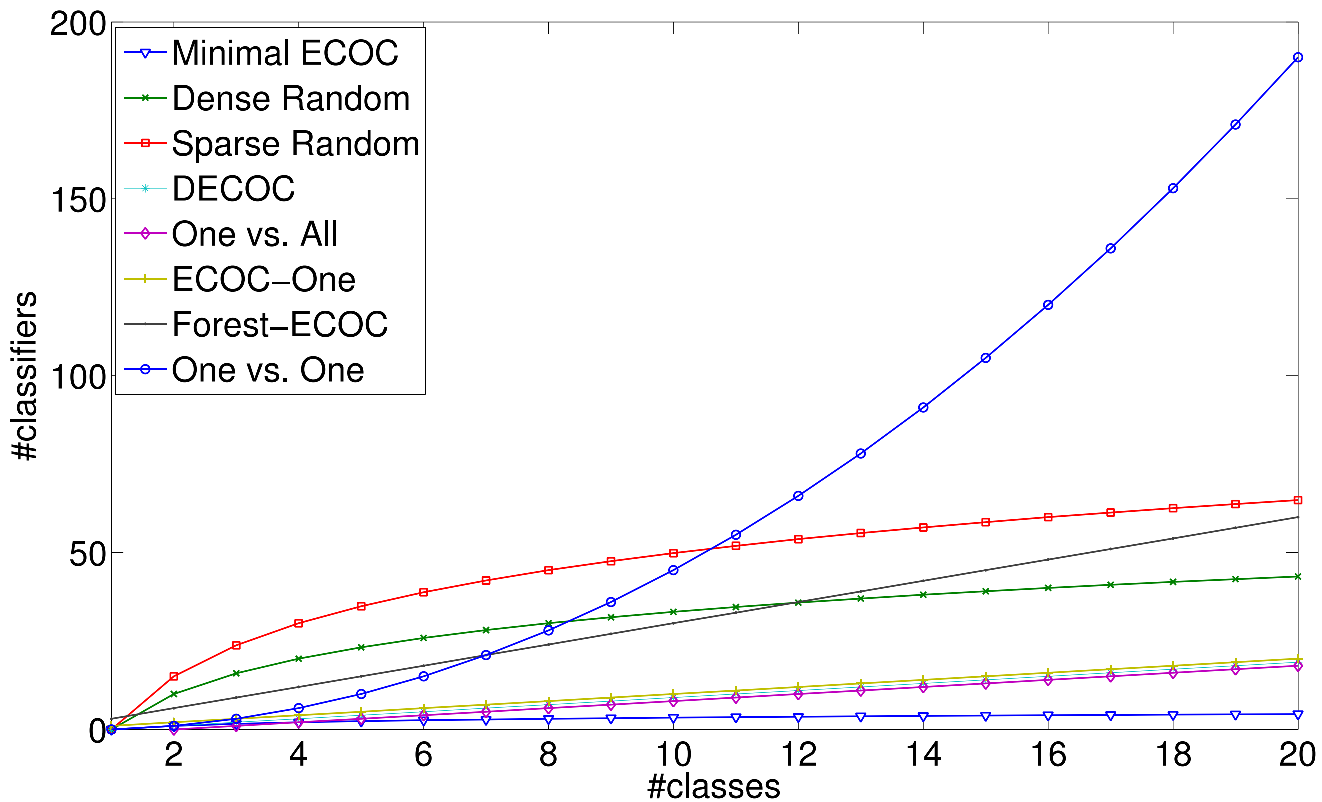
- **Error-Correction** is **extremely important:**
  - Number of binary classifiers that can miss without affecting the final prediction [1].

$$\frac{\min(\delta(\mathbf{m}^i, \mathbf{m}^j)) - 1}{2}, \forall_{i,j} \in \{1, \ldots, k\}$$

  - Measured as a single scalar [1].
  - Increasing it means that the number of classifiers (columns of ECOC matrix) will increase, and thus training complexity will increase.

[1] Dietterich, T. G., & Bakiri, G. (1995). Solving multiclass learning problems via error-correcting output codes. Journal of artificial intelligence research, 263-286.

- ECOC codings define a **super-linear** number of classifiers.
  - In this dissertation we are interested in **sub-linear** designs.
  - Given the reduce number of classifiers used by sub-linear designs, they should **exploit the multi-class data distribution**.



SUMMARY

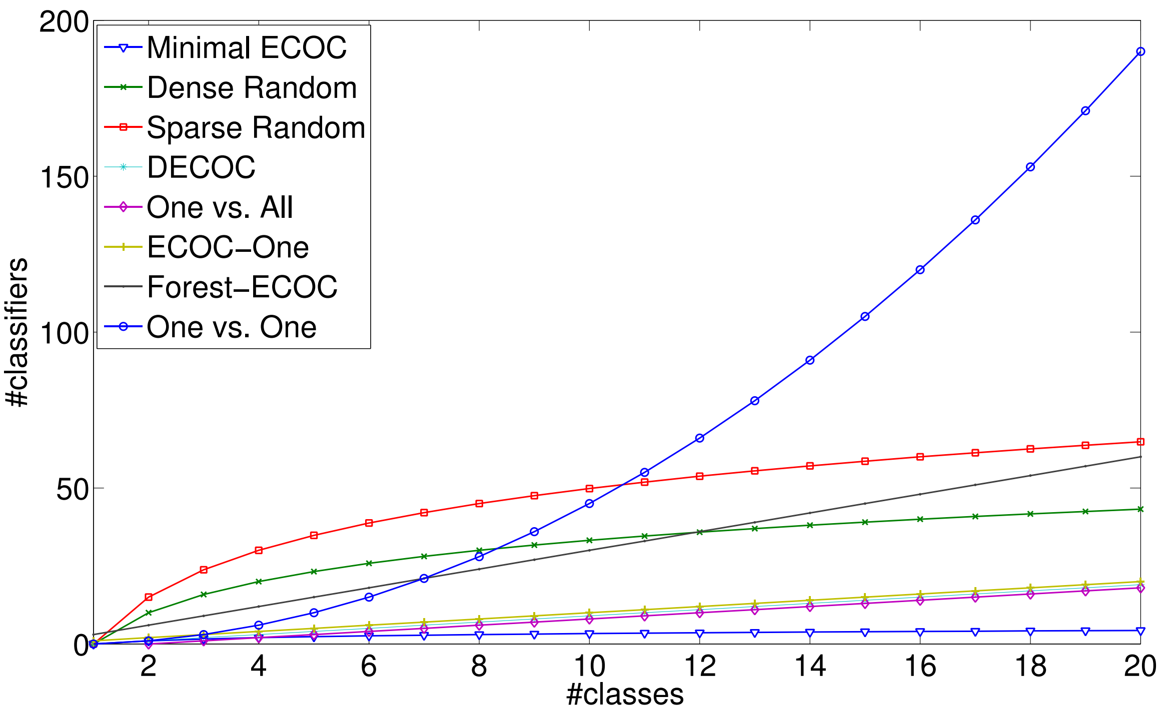
- ECOCs are a **powerful tool** to deal with multi-class problems**.**
- **Several coding designs** predefined, random and problem-dependent.
- Unlimited number of decoding designs.
- **Good practices of ECOC** are redefined as constraints.
- We define the ECOC of **Minimal length**.
- **Sub-linear ECOCs** should be optimized to compensate for the reduced number of classifiers.

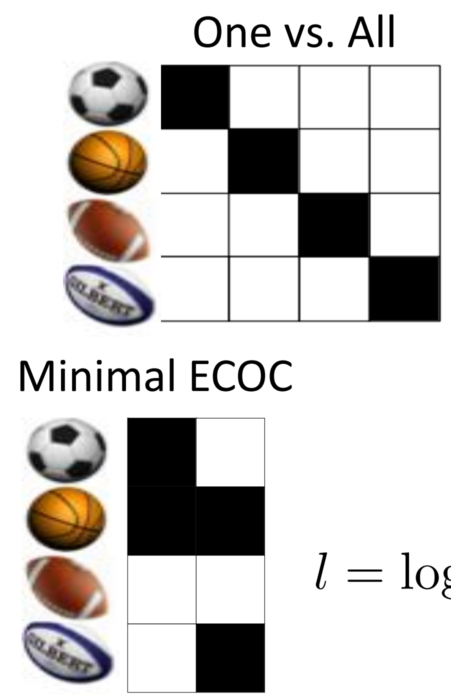
- Error-Correcting Output Codes
- **Learning ECOCs using Genetic Algorithms**
- Learning ECOC via Error-Correting Factorization
- Conclusions

Learning ECOCs using Genetic Algorithms (ECOC-GA)

- Exploit **multi-class data distribution** to find **sub-linear ECOC** designs with high performance.

- What is the **shortest code length** that can be defined [1,2]?





$$l = \log_2(k)$$

[1] Bautista, M. A., Baró, X., Pujol, O., Radeva, P., Vitrià, J., & Escalera, S. (2010). Compact evolutive design of error-correcting output codes. In Proceedings of the Supervised and Unsupervised Methods and their Applications (SUEMA), European Conference on Machine Learning and Principles and Practice of Knowledge Discovery in Databases.
[2] Bautista, M. Á., Escalera, S., Baró, X., Radeva, P., Vitriá, J., & Pujol, O. (2012). Minimal design of error-correcting output codes. Pattern Recognition Letters, 33(6), 693-702.

- Optimize the ECOC matrix given the data distribution and the classifiers is a **NP-complete problem** [1].
  - The ECOC search space is **extremely large**.

$$\binom{k}{2^l} \qquad \frac{2^l!}{2(2^l - k)!}$$

  - The search space is **not continuous** (ECOC coding matrices are discrete).
  - The search space is **not differentiable**.

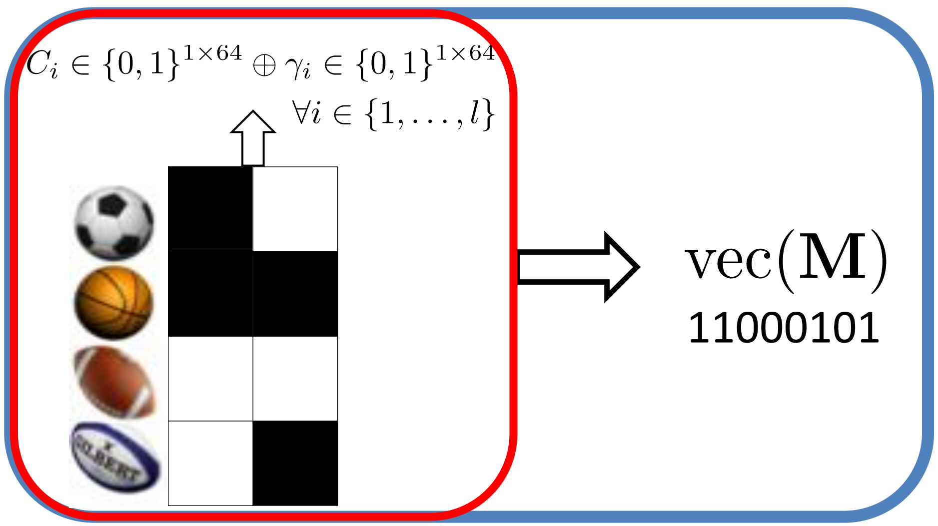
- **Genetic Algorithms** (GA) are often applied in this setting with **benefitial results**.

[1]Utschick, W., & Weichselberger, W. (2001). Stochastic organization of output codes in multiclass learning problems. Neural Computation, 13(5), 1065-1102. Chicago

- Genetic Algorithms are stochastic optimization methods based on **Darwin's Evolution** Theory [1].
- The fitness of indivudals is improved over generations by using **crossover and mutation operators**.
- **Our proposal [2,3]**:
  - **Optimize the ECOC** matrix using **Genetic Algorithms:**
    - Define how good an individual is (fitness function): classification accuracy.
    - Define how to represent a solution as a binary string (encoding): already binary.
  - **Optimize** the parameters of **binary classifiers** (SVM-RBF) using **Genetic Algorithms:**
    - Define how good a solution is: classification accuracy.
    - Define how to represent a solution as a binary string: binary representation of C, γ .
  - Use **standard crossover and mutation operators**

[1] Baluja, S., & Caruana, R. (1995, May). Removing the genetics from the standard genetic algorithm. In Machine Learning: Proceedings of the Twelfth International Conference (pp. 38-46).
[2] Bautista, M. A., Baró, X., Pujol, O., Radeva, P., Vitrià, J., & Escalera, S. (2010). Compact evolutive design of error-correcting output codes. In Proceedings of the, European Conference on Machine Learning Workshops.
[3] Bautista, M. Á., Escalera, S., Baró, X., Radeva, P., Vitriá, J., & Pujol, O. (2012). Minimal design of error-correcting output codes. Pattern Recognition Letters, 33(6), 693-702.

- An **ECOC individual** is represented as a **binary vector** and evaluated by means of its classification error.
- **Iterative** 2-step procedure:



1) Optimize the SVMs parameters.

2) Optimize the coding matrix and return to step 1.

- **Standard genetic operators** are used: scattered crossover and gaussian mutation:
  - Scattered crossover: randomly selects a set of points for each parent.
  - Gaussian mutation: adds a random number taken from a Gaussian distribution with mean 0 to each entry of the parent vector.

- We compare our proposal with Binary Minimal ECOC, PBIL [1] Minimal ECOC, One vs. All, One vs. One, Discriminant ECOC [2] and Forest-ECOC [2] approaches.
- Experimental settings:
  - We generated $10 \times k$ individuals per problem in the first generation.
  - We used SVMs with RBF kernel as our binary classifier.
    - Parameters were tuned using either GAs or PBIL for all methods, using two-fold cross-validation.
  - We used the Hamming Decoding distance.
  - We report the average classification accuracy over a stratified 10 fold-cross validation.
- A cache of dichotomizers is stored to leverage training time.

[1] Baluja, S. (1994). Population-based incremental learning. a method for integrating genetic search based function optimization and competitive learning (No. CMU-CS-94-163). Carnegie-Mellon Univ Pittsburgh Pa Dept Of Computer Science.

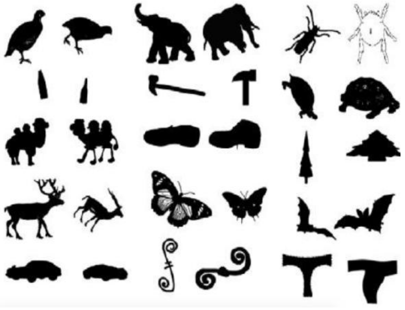
[2] Pujol, O., Radeva, P., & Vitria, J. (2006). Discriminant ecoc: A heuristic method for application dependent design of error correcting output codes. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 28(6), 1007-1012.

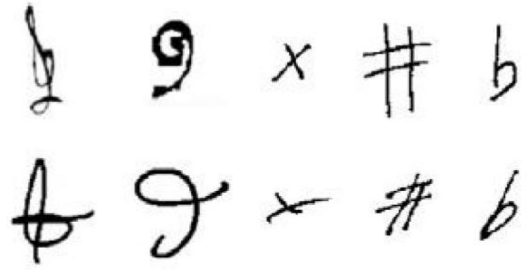
[3] Baró, X., Escalera, S., Vitrià, J., Pujol, O., & Radeva, P. (2009). Traffic sign recognition using evolutionary adaboost detection and forest-ECOC classification. Intelligent Transportation Systems, IEEE Transactions on, 10(1), 113-126.

- We evaluated experiments on 12 UCI datasets.

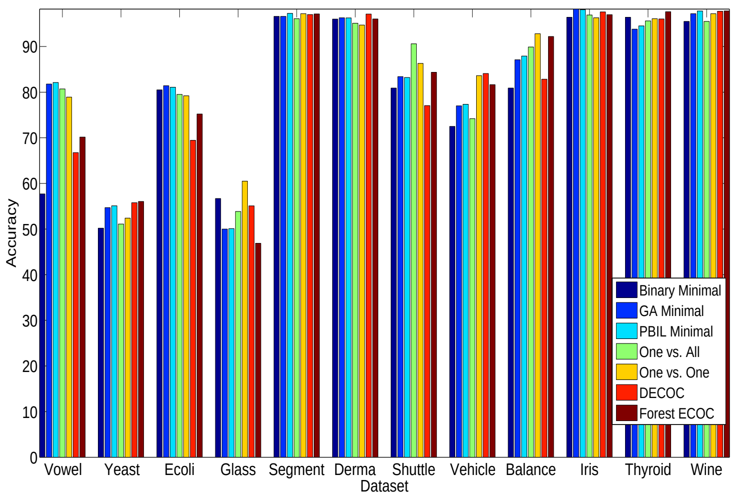
Problem	#Training samples	#Features	#Classes
Dermathology	366	34	6
Iris	150	4	3
Ecoli	336	8	8
Vehicle	846	18	4
Wine	178	13	3
Segmentation	2310	19	7
Glass	214	9	7
Thyroid	215	5	3
Vowel	990	10	11
Balance	625	4	3
Shuttle	14500	9	7
Satimage	4435	36	6
Yeast	1484	8	10

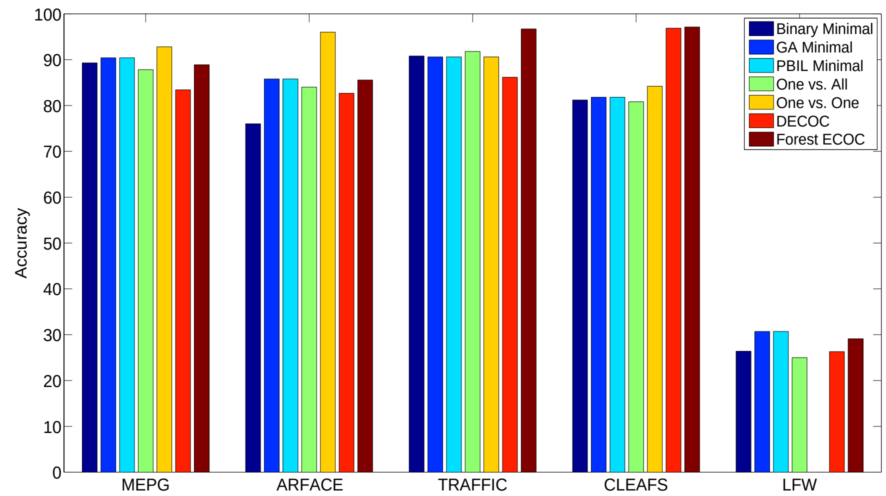
- We tackled 5 Computer Vision problems:
  - Labeled Faces in the wild: 610 categories
  - MPEG visual objects: 70 categories
  - Traffic sign categorization: 36 classes
  - ARFace dataset: 20 classes
  - Old music scores: 7 classes

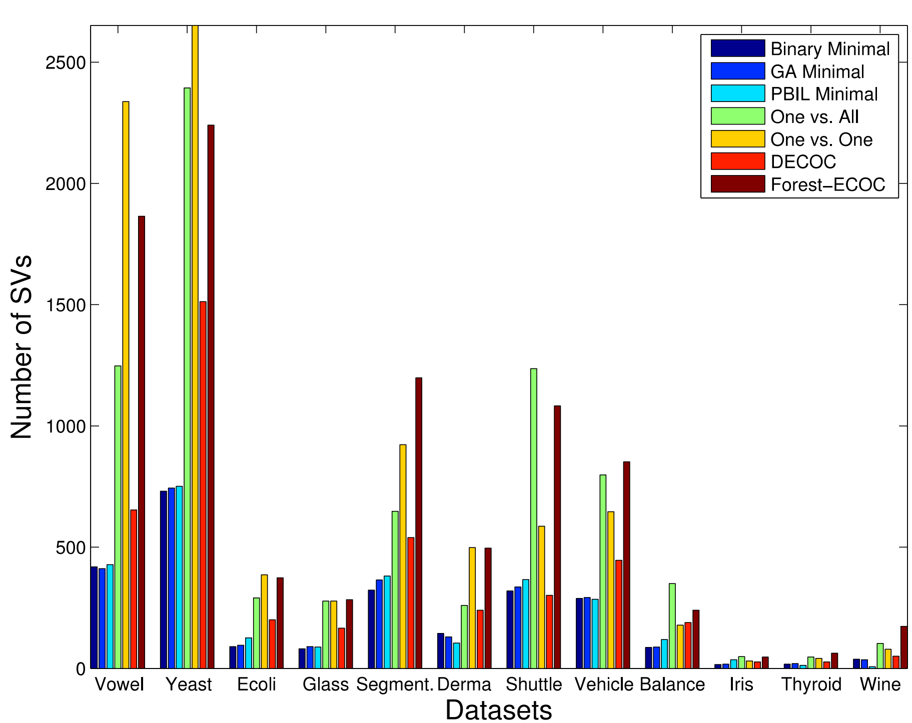
- Classification accuracies for each method.

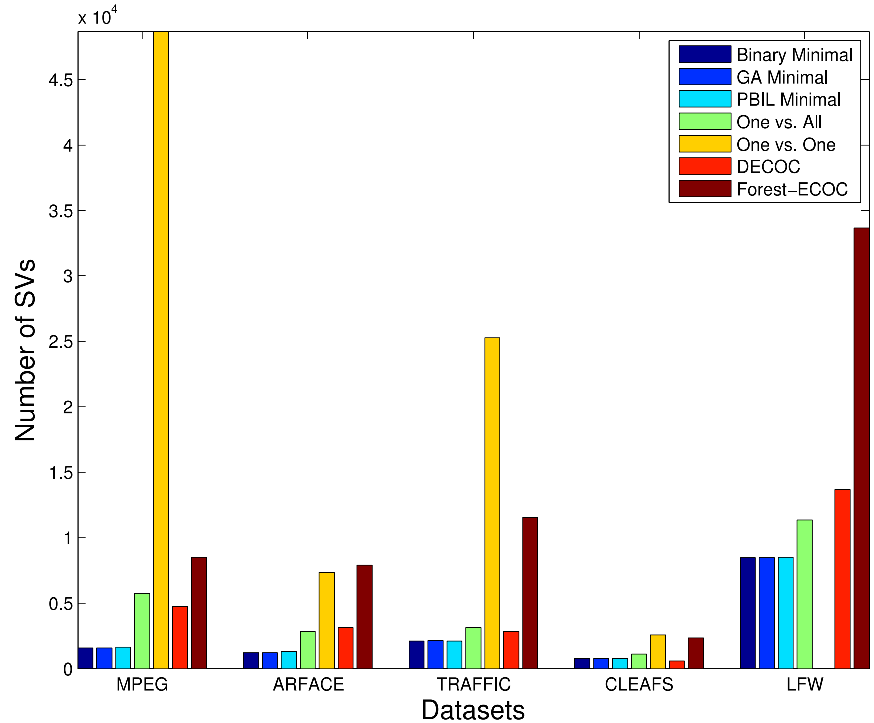




Binary Minimal	GA Minimal	PBIL Minimal	OVA	OVO	DECOC	Forest ECOC
5,2	3,6	3	4,8	3,7	4,2	3,1

- Number of Support Vectors for each method.





x2.9 over OVA

x3.2 over OVO

x1.8 over OVA

x7.4 over OVO

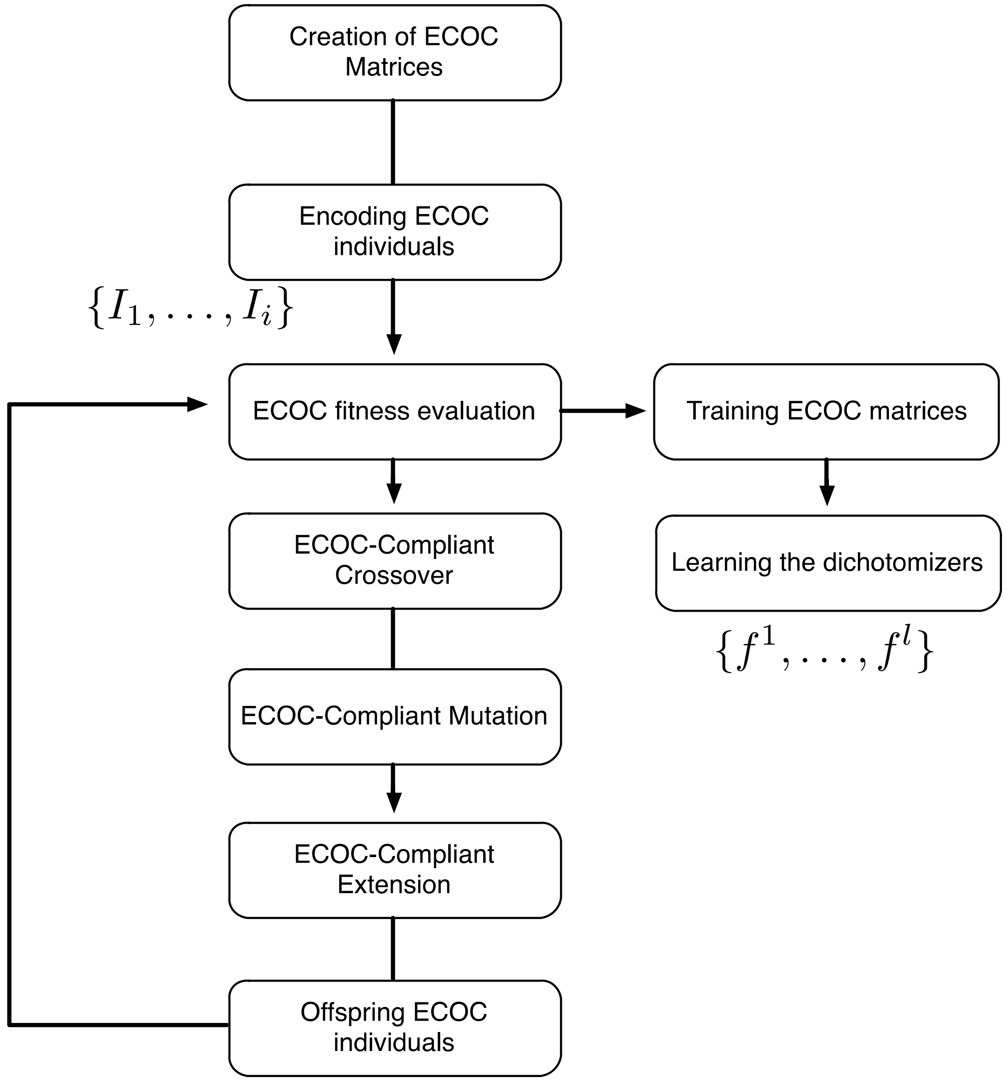
SUMMARY

- **Minimal ECOCs** are suitable for multi-class classification, when the **coding matrix is optimized** using Genetic Algorithms.
- With the **minimal number of classifiers** we obtain comparable or even better classification accuracies than state-of-the-art works.
- To obtain a high performance **we optimize the parameters of the binary classifiers** using **GAs**.
- **Large-scale tasks** can be tackled with **Minimal ECOCs**.

- Standard operators overlook **individuals structure**.
  - Generation of vast number of non-valid individuals makes the **algorithm ineffective**.

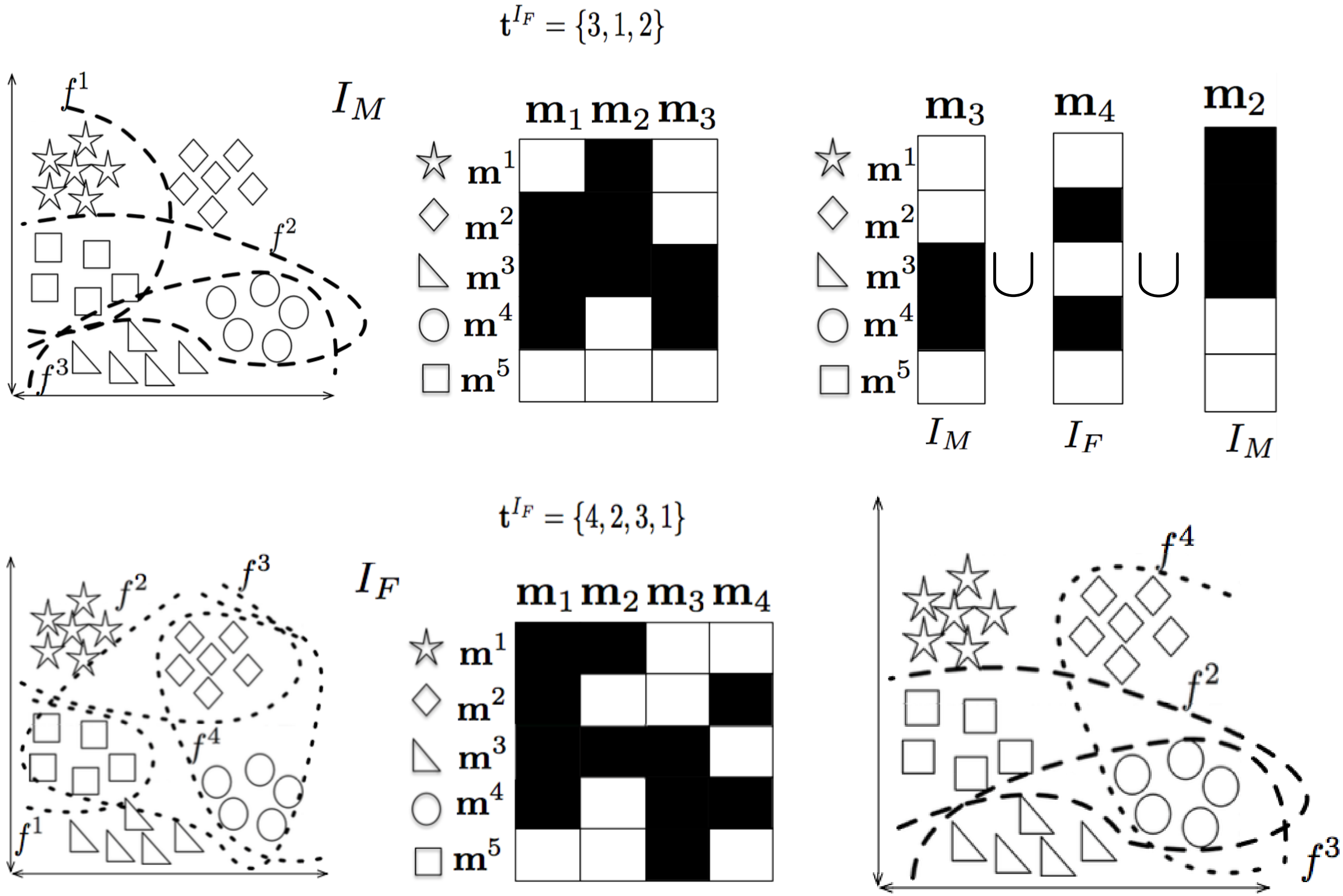
- **New proposal: ECOC-Compliant Genetic Algorithm**
  - Redefine **crossover and mutation** operators in order to take into account **ECOC properties**.
  - Possibility of including new operators?
    - Controlling the number of classifiers.
    - Adding and removing classifiers when needed.
  - Operators should be **fast and simple**.

- General pipeline for the **ECOC-Compliant Genetic Algorithm** [1].

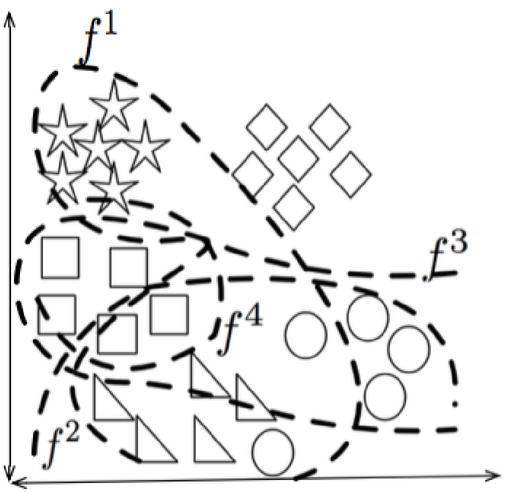


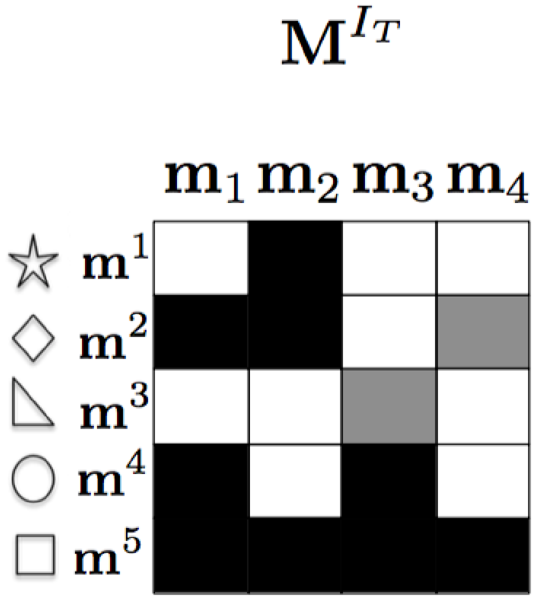
[1] Bautista, M. Á., Escalera, S., Baró, X., & Pujol, O. (2014). On the design of an ECOC-Compliant Genetic Algorithm. Pattern Recognition, 47(2), 865-884.

- ECOC-Compliant **crossover** operator.



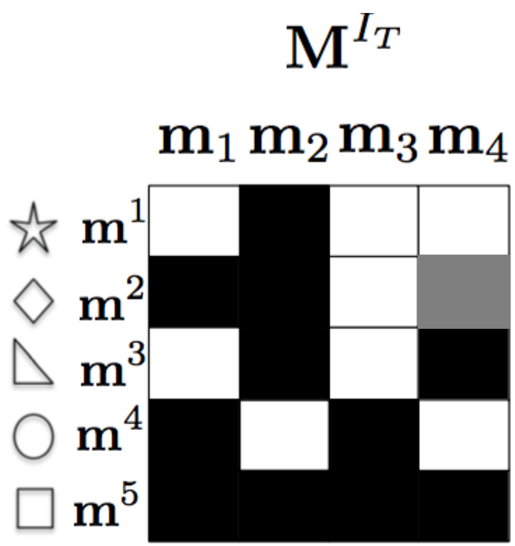
- ECOC-Compliant **mutation** operator.

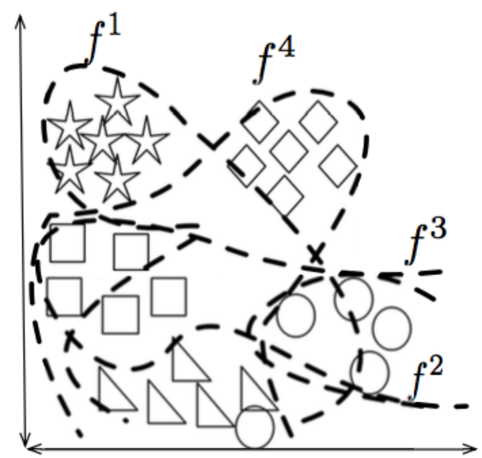




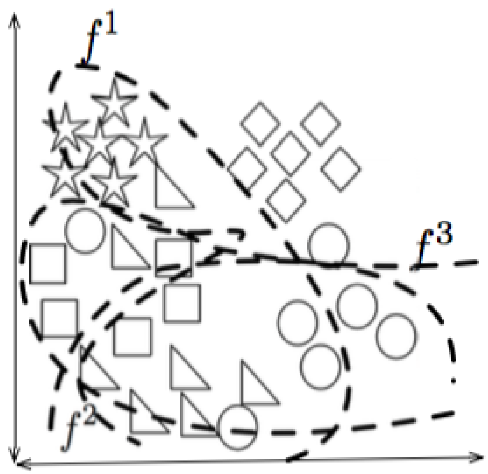
$\mathbf{C}^{I_T}$

	$\mathbf{c}^1$	$\mathbf{c}^2$	$\mathbf{c}^3$	$\mathbf{c}^4$	$\mathbf{c}^5$
$\mathbf{c}^1$	6	0	0	0	0
$\mathbf{c}^2$	0	6	0	0	0
$\mathbf{c}^3$	0	0	3	1	0
$\mathbf{c}^4$	0	0	2	5	2
$\mathbf{c}^5$	0	0	0	0	3

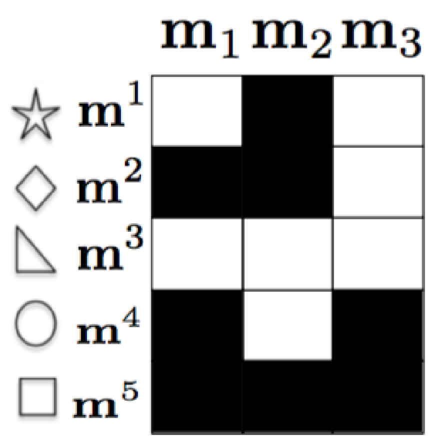




- ECOC-Compliant **extension** operator.



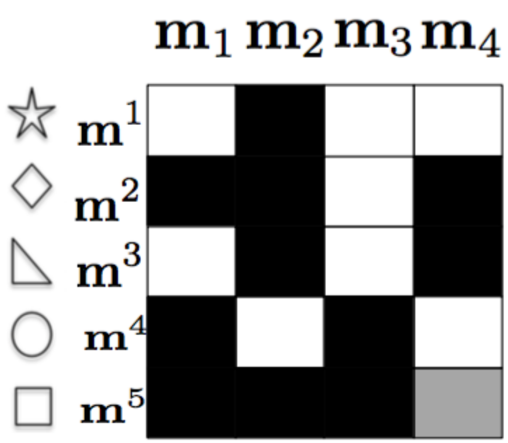
$\mathbf{M}^{I_T}$

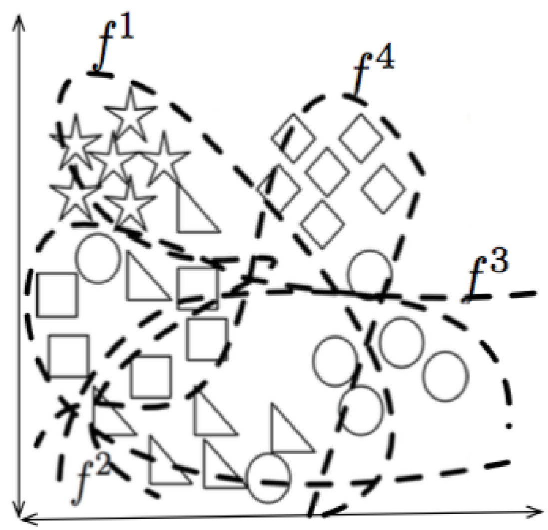


$\mathbf{C}^{I_T}$

	$\mathbf{c}^1$	$\mathbf{c}^2$	$\mathbf{c}^3$	$\mathbf{c}^4$	$\mathbf{c}^5$
$\mathbf{c}^1$	6	0	1	0	0
$\mathbf{c}^2$	0	6	0	1	0
$\mathbf{c}^3$	0	0	3	1	0
$\mathbf{c}^4$	0	0	2	5	2
$\mathbf{c}^5$	0	0	1	1	3

$\mathbf{M}^{I_T}$

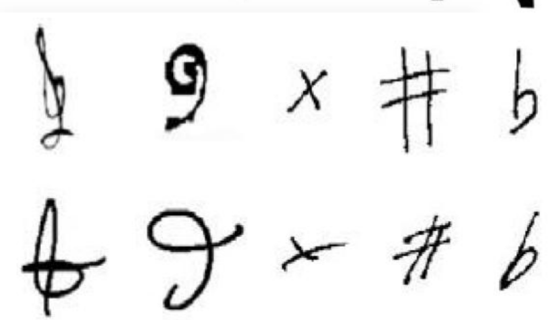




- We run experiments on 9 UCI datasets.

Problem	#Training samples	#Features	#Classes
Vowel	990	10	11
Yeast	1484	8	10
Ecoli	336	8	8
Glass	214	9	7
Segmentation	2310	19	7
Dermathology	366	34	6
Shuttle	14500	9	7
Vehicle	846	18	3
Satimage	4435	36	6

- We tackled 4 Computer Vision problems:
  - MPEG visual objects: 70 categories
  - Traffic sign categorization: 36 classes
  - ARFace dataset: 20 classes
  - Old music scores: 7 classes



- We compare our proposal with Binary Minimal ECOC, Lorena et. al [1], Pedrajas et. al [2], One vs. All, One vs. One, Discriminant ECOC and Forest-ECOC approaches.

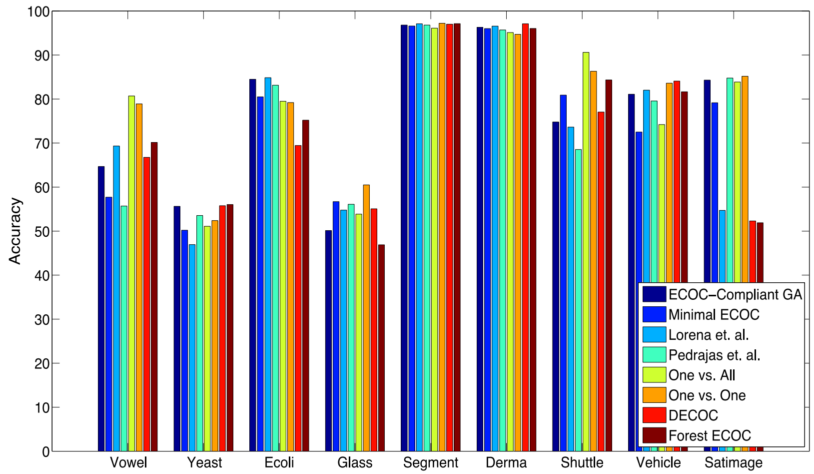
- Experimental settings:
  - We generated $10 \times k$ individuals per problem.
  - We used SVMs with RBF kernel as our binary classifier.
    - Parameters were tuned using Gas for all methods, using two-fold cross-validation.
  - We used the Loss-Weighted decoding.
  - We report the average classification accuracy over a stratified 10 fold-cross validation.

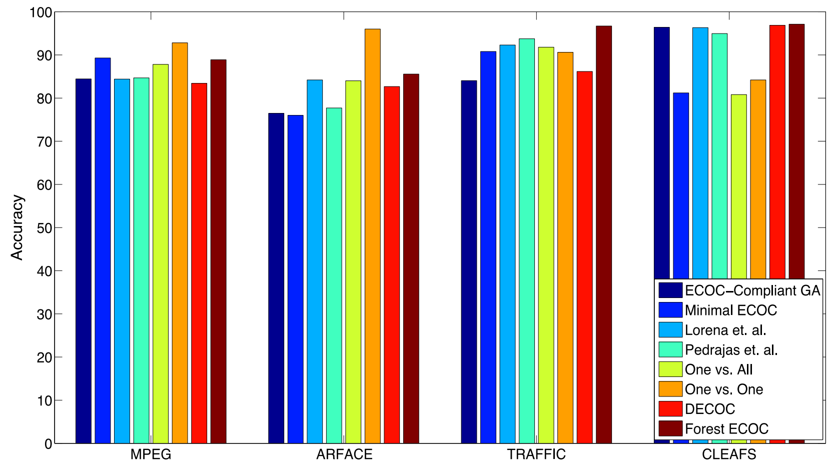
- A cache of dichotomizers is stored to leverage training time.

[1] Lorena, A. C., & de Carvalho, A. C. (2006, October). Multiclass SVM design and parameter selection with genetic algorithms. In Neural Networks, 2006. SBRN'06. Ninth Brazilian Symposium on (pp. 131-136). IEEE. Chicago Lorena, Ana Carolina, and André CPLF d

[2] Garcia-Pedrajas, N., & Fyfe, C. (2008). Evolving output codes for multiclass problems. Evolutionary Computation, IEEE Transactions on, 12(1), 93-106.

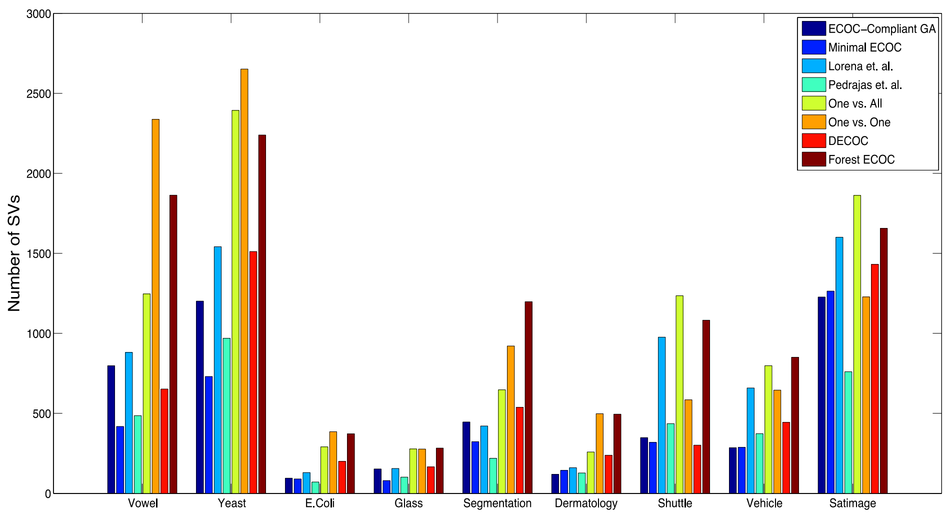
- Classification accuracies for each method on UCI datasets.

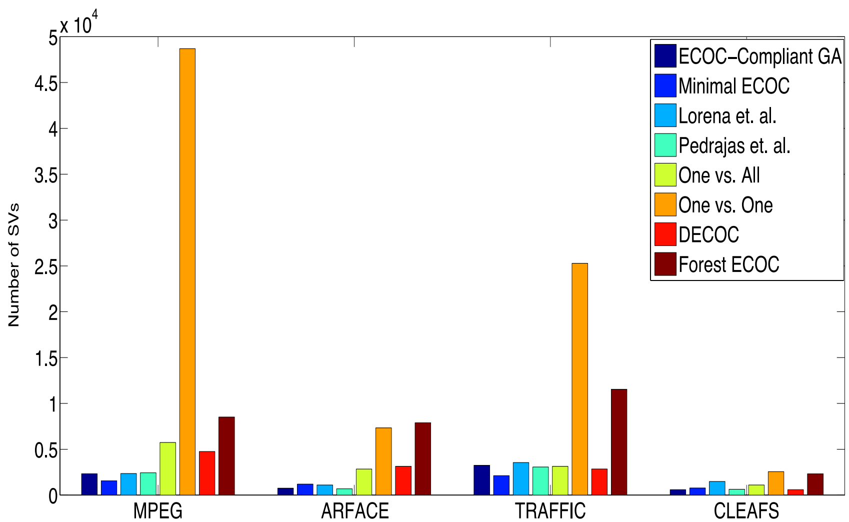




ECOC-Compliant GA	Minimal ECOC	Lorena et. al	Pedrajas et. al	OVA	OVO	DECOC	Forest ECOC
4.5	6.2	4.4	4,8	4.7	3.2	4.3	3.4

- Number of support vectors for each method on UCI datasets.





x1.9 over OVA

x2.0 over OVO

x1.8 over OVA

x12.0 over OVO

SUMMARY

- We propose to **redefine the operators** to take into account the **properties of Error-Correcting Output codes.**
- The novel genetic operators **avoid the generation of non-valid individuals**.
- To obtain a high performance we optimize the parameters of the binary classifiers SVM-RBF using GA.
- Results show that we obtain comparable or even better results than state-of-the-art ECOC design while **reducing drastically the number of Support Vectors.**

CONCLUSIONS

- **Genetic Algorithms** can be powerful tools to **optimize ECOC coding matrices**.
- Avoid the generation of non-valid individuals.
- The training computational cost can be leveraged using simple speed up tricks.
- The **Error-Correcting properties** of an ECOC **cannot be exploited by the proposed GAs**.

Error-Correcting Output Codes

Learning ECOCs using Genetic Algorithms

Learning ECOC via Error-Correting Factorization

Conclusions

Learning ECOCs via Error-Correcting Factorization (ECOC-ECF)

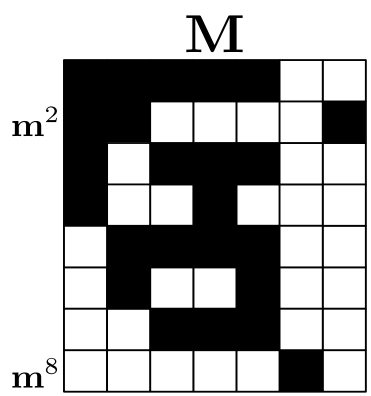
- In this dissertation we aim to deepen into open questions which call for further study of **Error-Correction capabilities of ECOCs**:
  1. How do **Minimal ECOC matrices** behave? ☑
  2. Is Error-Correction distributed evenly on all classes? ☒
  3. Can problem-dependent designs profit from the distribution of Error-Correcting capabilities? ☒
  4. Is it better to allocate Error-Correction to classes prone to error or to classes not prone to error? ☒
  5. Is there a problem-dependent definition of the minimum number of classifiers needed for an ECOC matrix? ☒

- **Error-Correction capability** of an ECOC **is key**:
  - It measures the **number of binary classifiers that can miss** its prediction without affecting the final multi-class prediction.

$$\frac{\min(\delta(\mathbf{m}^i,\mathbf{m}^j))-1}{2}$$

  - The correcting capability is measured as a **single scalar** for the whole ECOC matrix in state-of-the-art works.
  - A single **scalar does not provide information** about how the Error-Correction is distributed.
  - We can compute the **pair-wise distance matrix between codewords,** aka the Separability Matrix.

$$\mathbf{H} \in \mathbb{R}^{k\times k}$$



$$\lfloor \frac{\min(\mathbf{H})-1}{2} \rfloor = 0$$

H

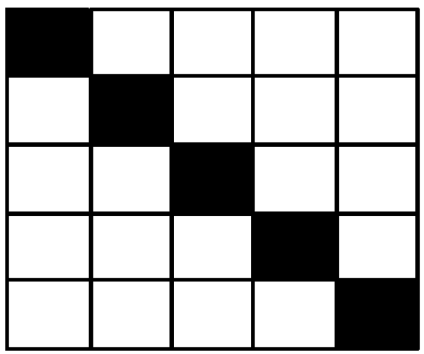
	0	4	1	3	1	3	2	6
$\mathbf{h}^2$	4	**0**	5	3	5	3	6	4
	1	5	**0**	2	2	4	1	5
	3	3	2	**0**	4	4	3	3
	1	5	2	4	**0**	2	1	5
	3	3	4	4	2	**0**	3	3
	2	6	1	3	1	3	**0**	4
$\mathbf{h}^8$	6	4	5	3	5	3	4	**0**

$$\lfloor \frac{\min(\mathbf{h}^2 \oplus \mathbf{h}^8)-1}{2} \rfloor = 1$$

- Analyzing the **pair-wise distance** enables a **deeper understanding** of how the **correction** is distributed among classes (how separated codewords are).

- Every ECOC matrix has its pair-wise distance matrix.

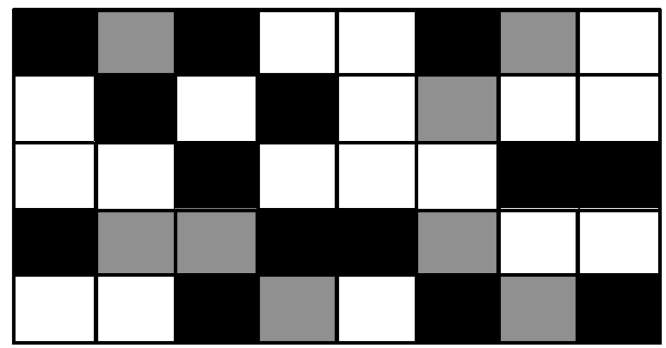
M



H

0	2	2	2	2
2	0	2	2	2
2	2	0	2	2
2	2	2	0	2
2	2	2	2	0

M

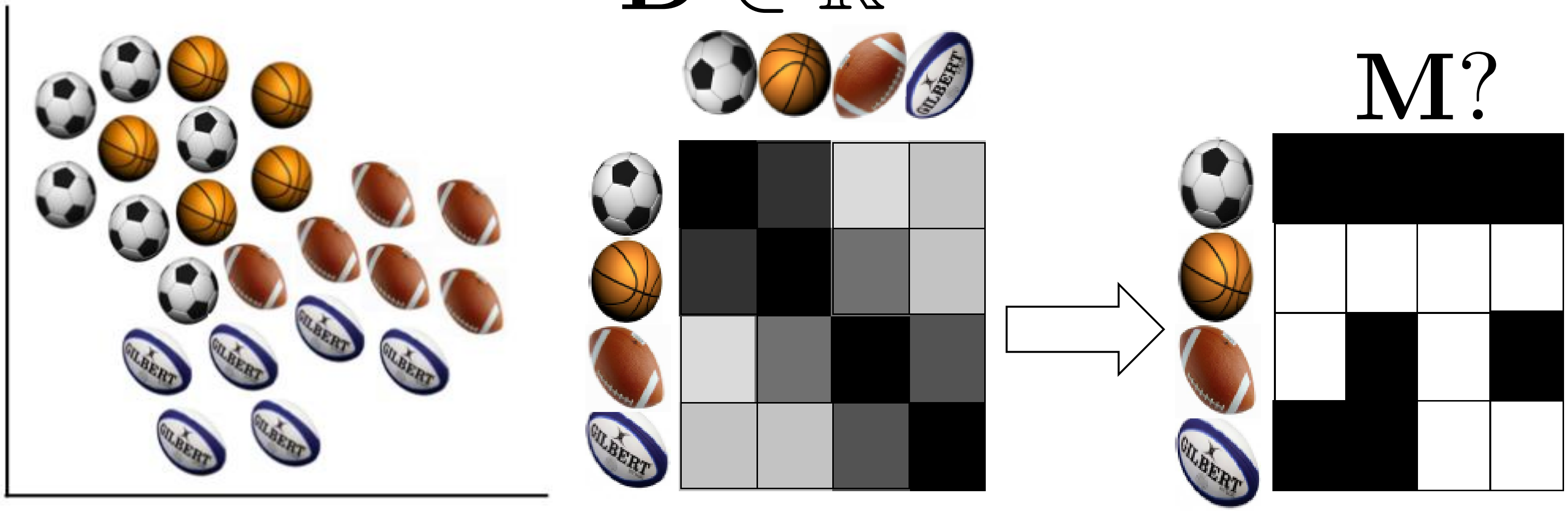


H

0	4	1	3	2
4	0	5	1	4
1	5	0	4	1
3	1	4	0	1
2	4	1	1	0

- Distance (Design) matrix → ECOC?
  - We define $\mathbf{D}$ as a design matrix that encodes the distances between pairs of codewords of a desired ECOC.

$$\mathbf{D} \in \mathbb{R}^{k \times k}$$



  - Extract $\mathbf{D}$ from multi-class data is easy (e.g heuristiscs like Mahalanobis distances between classes.)
  - Information of experts can be easily coded.
  - Analyzing $\mathbf{D}$ can assist to solve the number of classifiers problem.

- **Motivation:**
  - Is it possible to find an ECOC such that it follows the distances denotes in a design matrix?
  - What conditions should hold for an ECOC to encode an arbitrary design matrix?

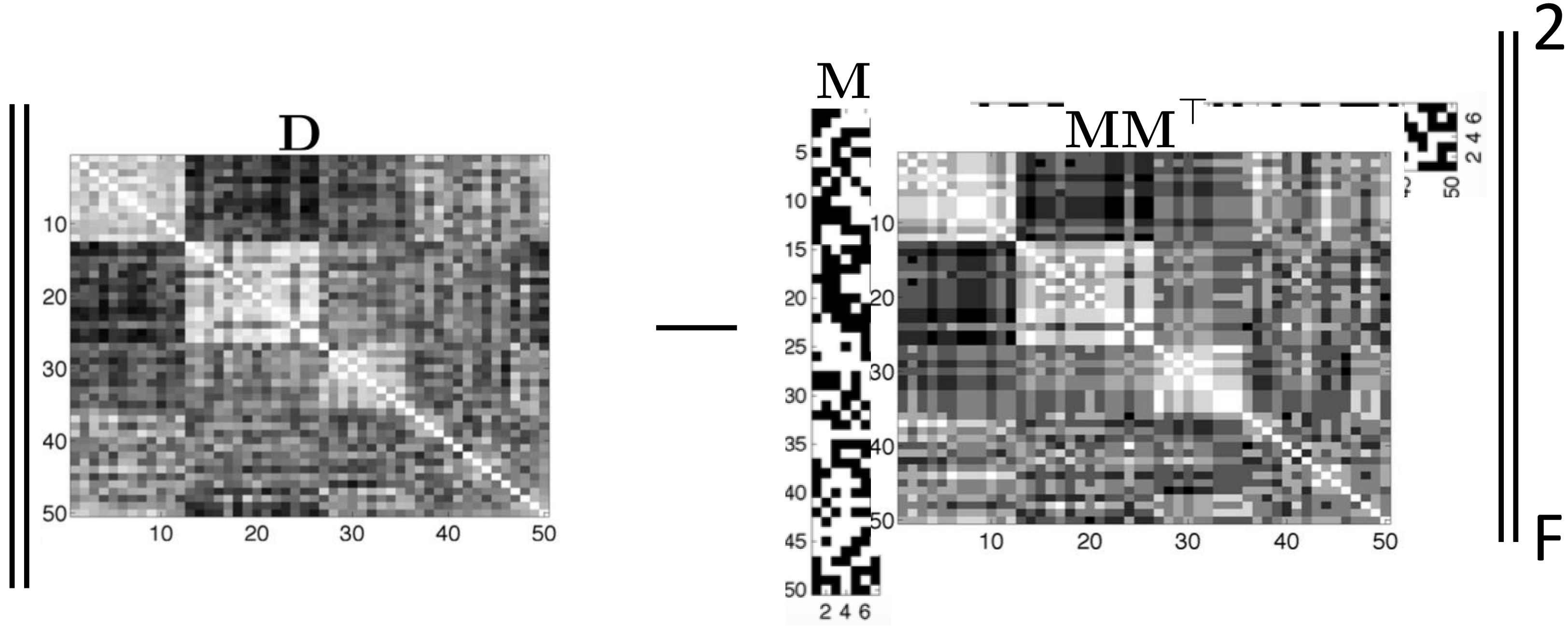
$$h_{ij} < h_{kl} \iff d_{ij} < d_{kl} \forall_{i,j,k,l}$$

  - However the l_1 distance can be seen as:

$$\|\mathbf{m}^i - \mathbf{m}^j\|_1 = \frac{-(\mathbf{m}^i {\mathbf{m}^j}^\top) + l}{2} \quad \mathbf{m} \in \{-1, +1\}$$

  - We can work with the **inner product** equivalently.
  - Factorize the design matrix!

- A visualization of the problem.



- What is the **number of classifiers** needed to **minimize the norm**?

$$\mathrm{rank}(\mathbf{D}) = \mathrm{rank}(\mathbf{M}\mathbf{M}^\top) = \mathrm{rank}(\mathbf{M})$$

- Formulating the **Error-Correcting Factorization** (ECF) [1]:

$$
\begin{aligned}
\underset{\mathbf{M}}{\text{minimize}} \quad & \|\mathbf{D} - \mathbf{M}\mathbf{M}^\top\|_F^2 \\
\text{subject to} \quad & \mathbf{M} \in \{-1,+1\}^{k \times l} \\
& \mathbf{M}\mathbf{M}^\top - \mathbf{P} \leq 0 \\
& \mathbf{M}^\top\mathbf{M} - \mathbf{1}(l-1) \leq 0 \\
& -\mathbf{M}^\top\mathbf{M} - \mathbf{1}(l-1) \leq 0
\end{aligned}
$$

- **Non-convex!**
  - Quadratic term makes the objective function non-convex.
  - Discrete constraint makes the problem NP-Complete.
- **Good news:**
  - Discrete constraint can be relaxed $\mathbf{M} \in [-1,+1]$
  - Coordinate Descent has been successfully applied in non-convex problems (convergence to stationary points if problems are uniquely solved) [2].

[1] Bautista, M. A., Pujol, O., de la Torre, F., & Escalera, S. (2015). Error-Correcting Factorization. arXiv preprint arXiv:1502.07976. Under review at TPAMI
[2] Grippo, L., & Sciandrone, M. (2000). On the convergence of the block nonlinear Gauss–Seidel method under convex constraints. Operations Research Letters, 26(3), 127-136.

- **Codeword descent approach** for ECF[1]:
  - Optimize **the *i-th* codeword** of M while **fixing the rest** of the rows.

$$\begin{aligned} \underset{\mathbf{m}^i}{\text{minimize}} \quad & \left\| \begin{bmatrix} l & \mathbf{d}_i \\ \mathbf{d}^{iT} & \mathbf{D}'^i_{/i} \end{bmatrix} - \begin{bmatrix} \mathbf{m}^i \mathbf{m}^{iT} & \mathbf{M}'^i \mathbf{m}_i \\ \mathbf{M}'^i \mathbf{m}^{iT} & \mathbf{M}'^i \mathbf{M}'^{i\top} \end{bmatrix} \right\|_F^2 \\ \text{subject to} \quad & \mathbf{m}^i \in [-1, +1]^l \\ & \begin{bmatrix} \mathbf{m}^i \mathbf{m}^{iT} & \mathbf{M}'^i \mathbf{m}_i \\ \mathbf{M}'^i \mathbf{m}^{iT} & \mathbf{M}'^i \mathbf{M}'^{i\top} \end{bmatrix} - \begin{bmatrix} l & \mathbf{p}_i \\ \mathbf{p}^{iT} & \mathbf{P}'^i_{/i} \end{bmatrix} \leq 0. \end{aligned}$$

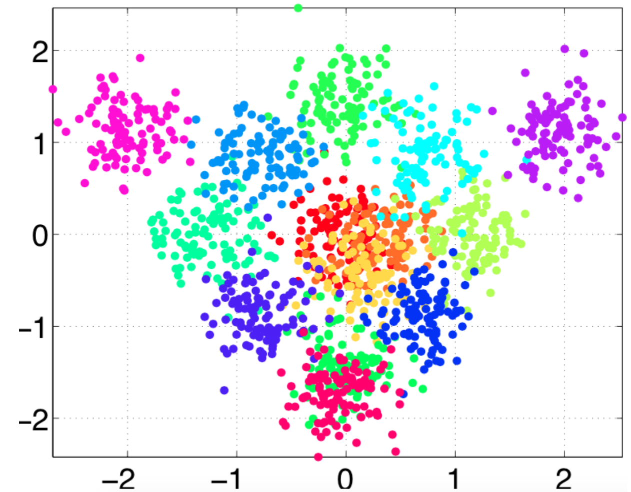
  - Terms not involve the *i-th* codeword can be drop, **reducing the ECF** problem to **least squares**:

$$\begin{aligned} \underset{\mathbf{m}^i}{\text{minimize}} \quad & \left\| \mathbf{M}'^i \mathbf{m}^i - \mathbf{d}^i \right\|_2^2 \\ \text{subject to} \quad & -1 \leq \mathbf{m}^i \leq +1 \\ & \mathbf{M}'^i \mathbf{m}^i - \mathbf{p}^i \leq 0 \end{aligned}$$

  - Least-squares can be **solved uniquely** (when not overdetermined), thus the algorithm is **guaranteed to converge** to **stationary point**.

[1] Bautista, M. A., Pujol, O., de la Torre, F., & Escalera, S. (2015). Error-Correcting Factorization. arXiv preprint arXiv:1502.07976. Under review at TPAMI

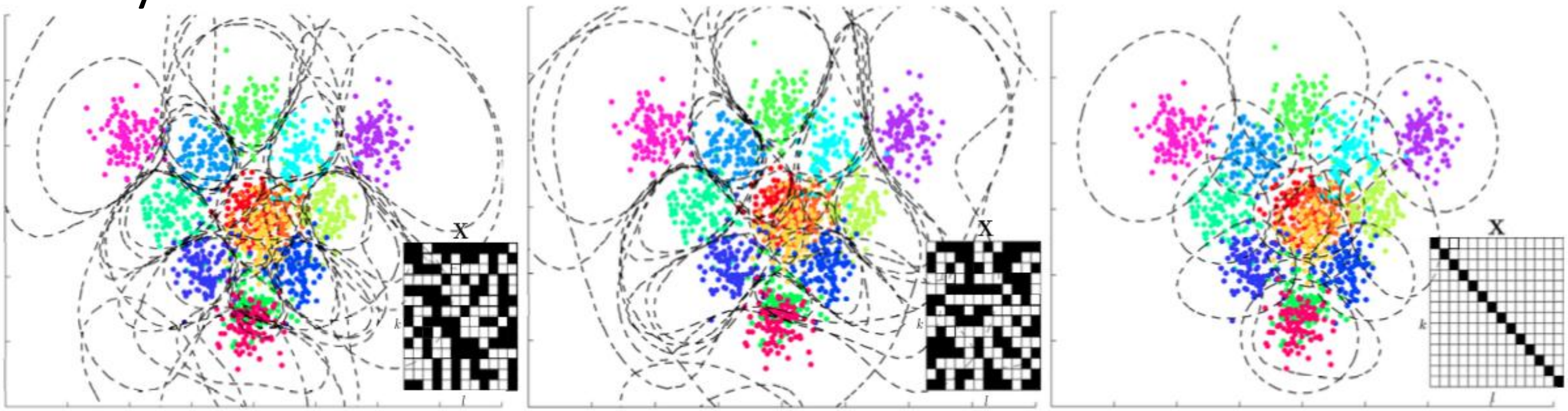
- We tested ECF on 8 UCI datasets and a Toy (synthetic) problem of 14 classes.



	Glass	Segment.	Ecoli	Yeast	Vowel	Toy	Traffic	ARFace
#s	214	2310	336	1484	990	400	3481	1300
#f	9	19	8	8	10	2	100	120
#c	7	7	8	10	11	14	36	50

- We also choose 2 Vision problems:
  - Traffic sign categorization: **36 classes**
  - ARFace dataset: **20 classes**

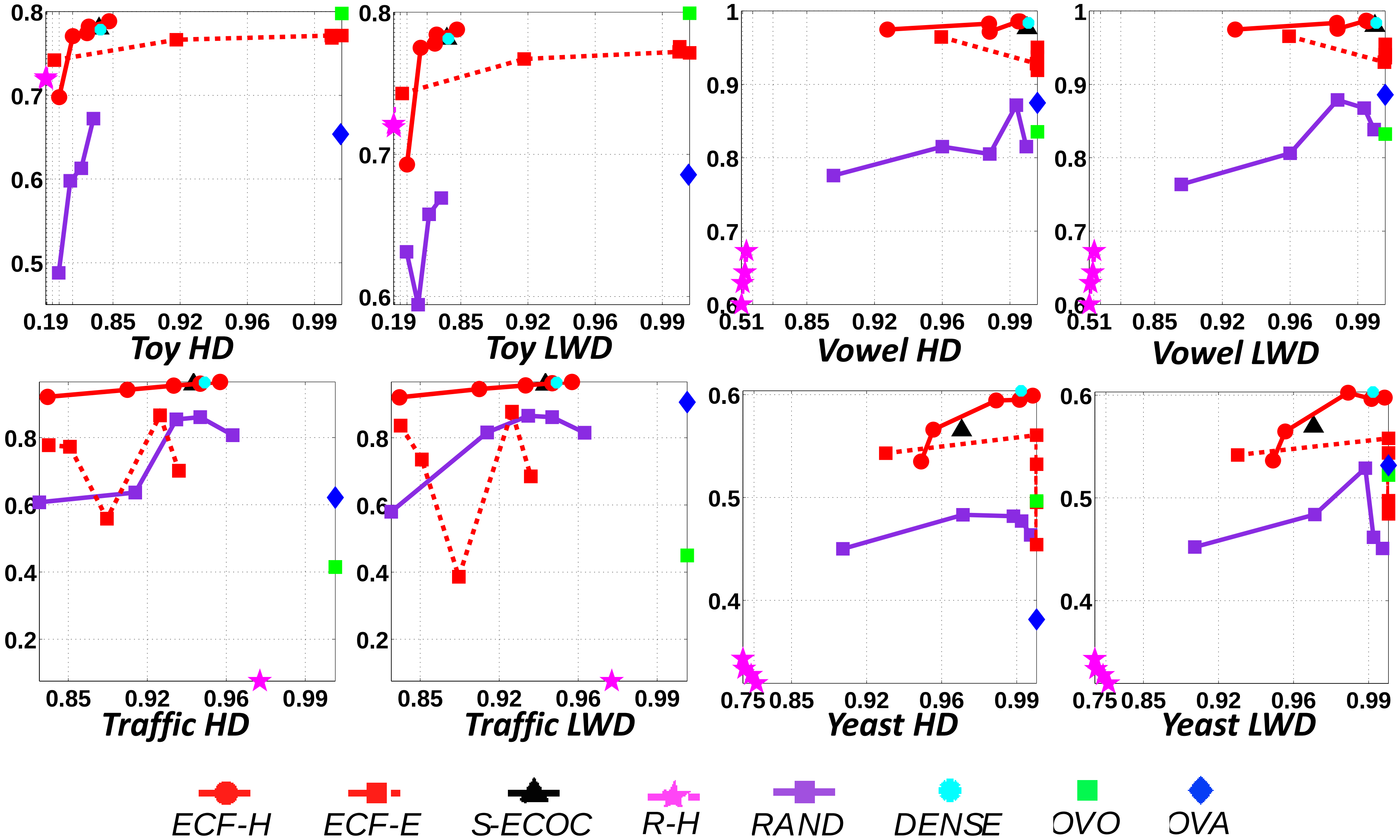
- Methods:
  - **ECF-H, where Error-Correction is allocated to classes prone to error.**
  - **ECF-E: Error-Correction is allocated to classes not prone to error.**
  - OVA, OVO, Random ECOC (RAND), Dense Random (DR), Spectral ECOC (S-ECOC)[1], Relaxed Hierarchy (R-H) [2].
- Binary classifier:
  - SVM-RBF with parameters optimized using grid-search.
  - Reported accuracy are the average over a 10-fold stratified cross-validation.
- Toy Problem



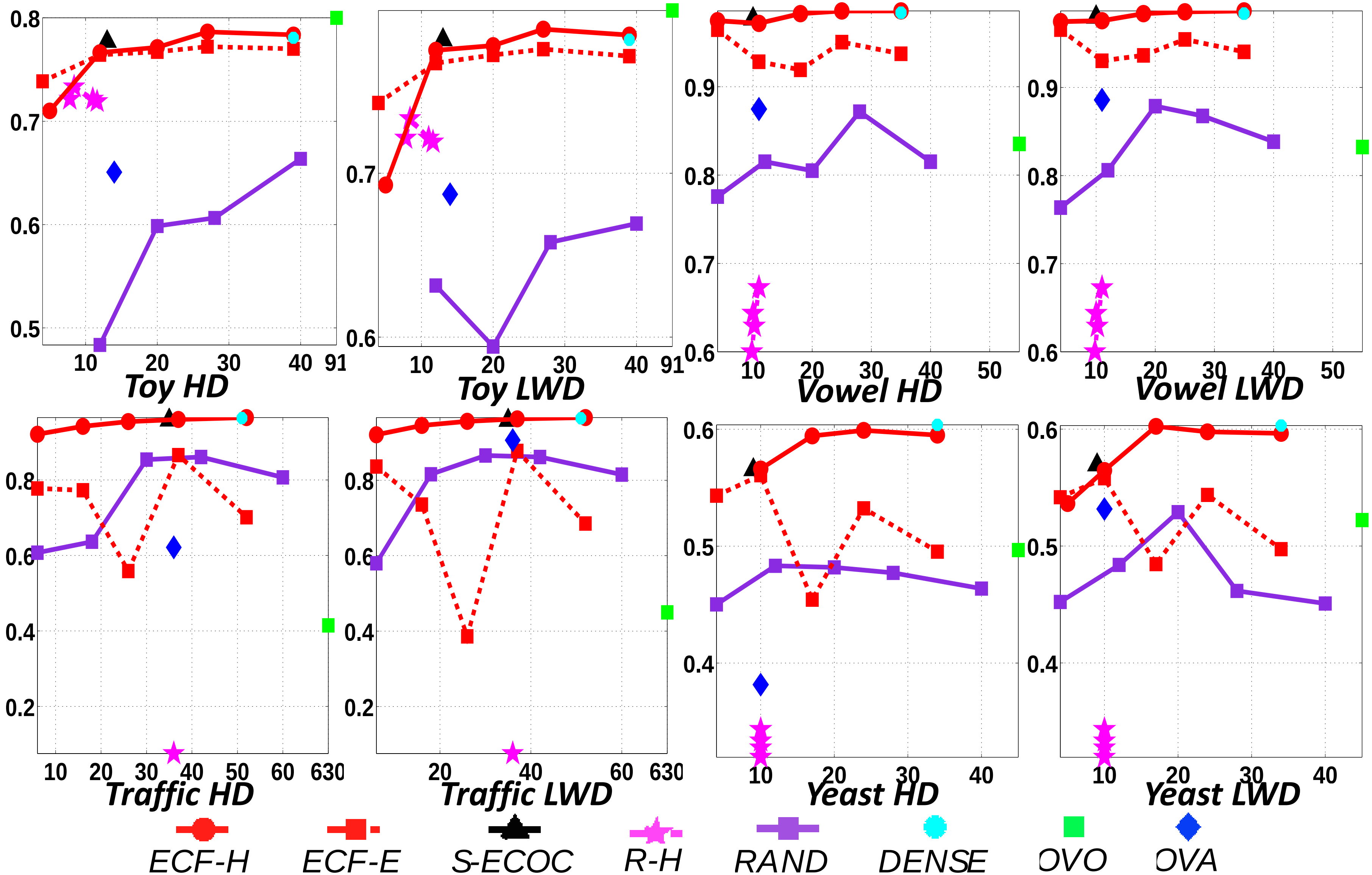
Error-Correcting Factorization 77.49% | Dense Random 66.45% | One versus All 49.53%

[1] Zhang, X., Liang, L., & Shum, H. Y. (2009, September). Spectral error correcting output codes for efficient multiclass recognition. In Computer Vision, 2009 IEEE 12th International Conference on (pp. 1111-1118). IEEE.
[2] Gao, T., & Koller, D. (2011, November). Discriminative learning of relaxed hierarchy for large-scale visual recognition. In Computer Vision (ICCV), 2011 IEEE International Conference on (pp. 2072-2079). IEEE.

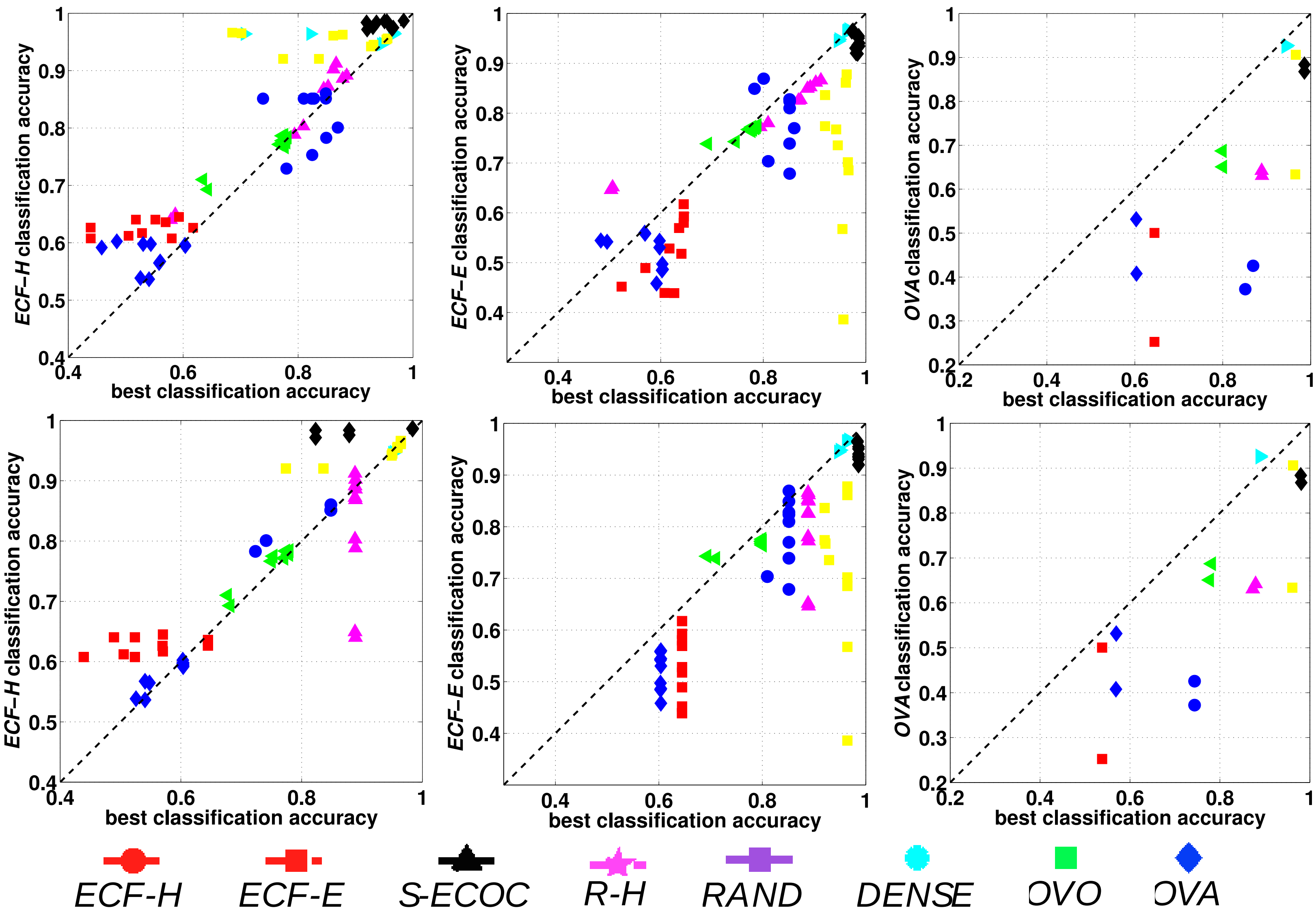
- Classification accuracy as a function of the relative complexity.



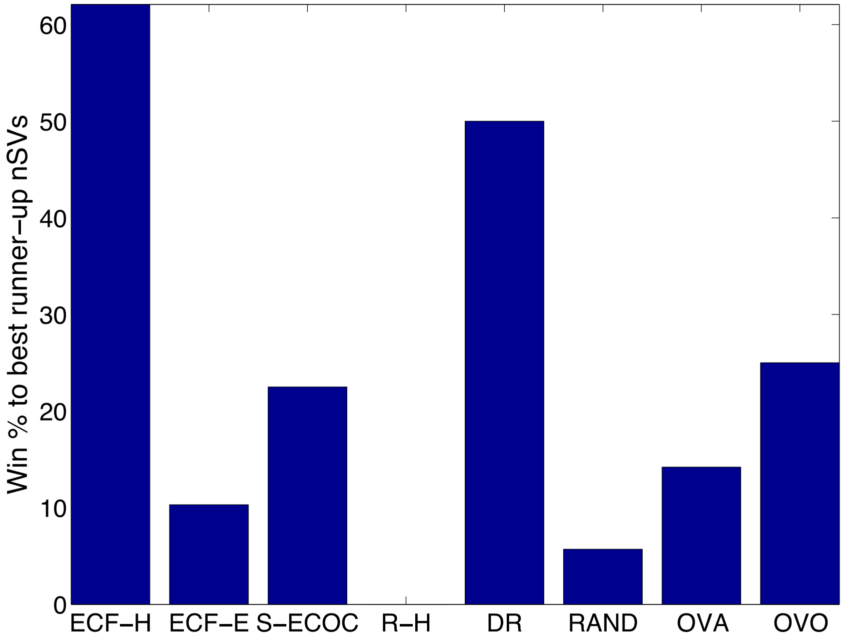
- Classification accuracy as a function of the number of classifiers.

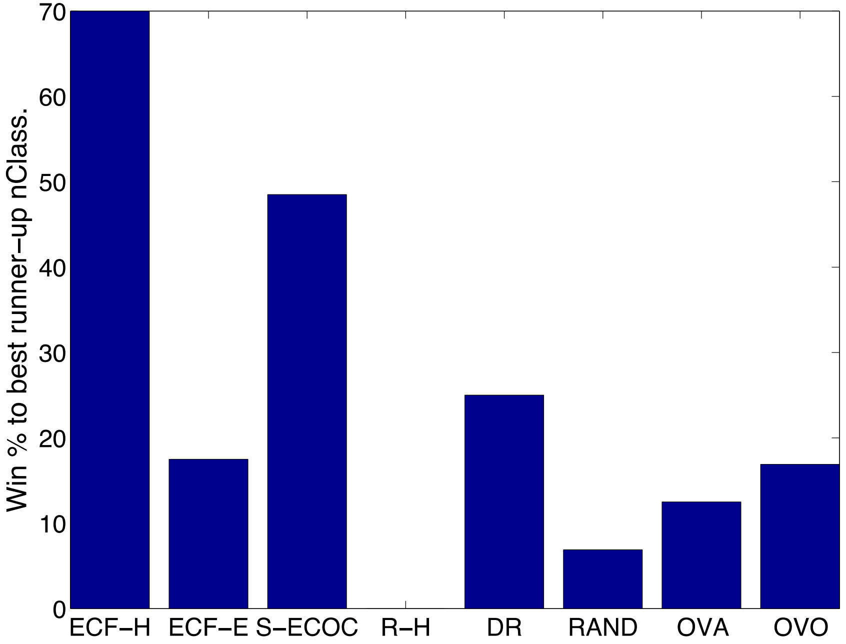


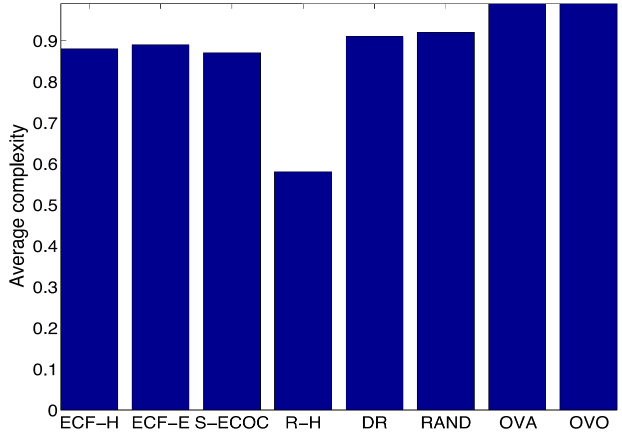
- Comparing ECF-H, ECF-E and OVA with the top performer.



- Proportion of times a method is the top performer.

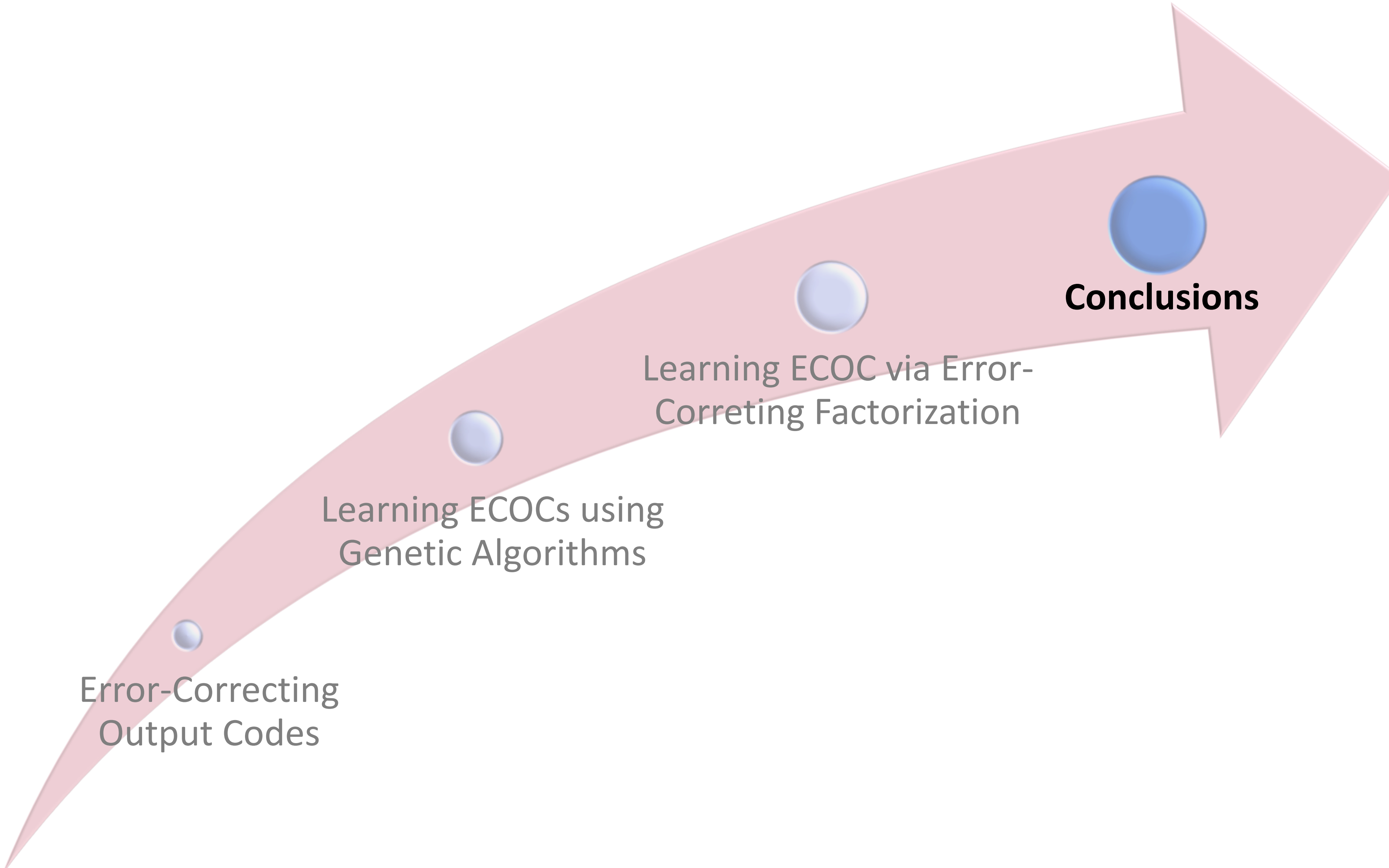






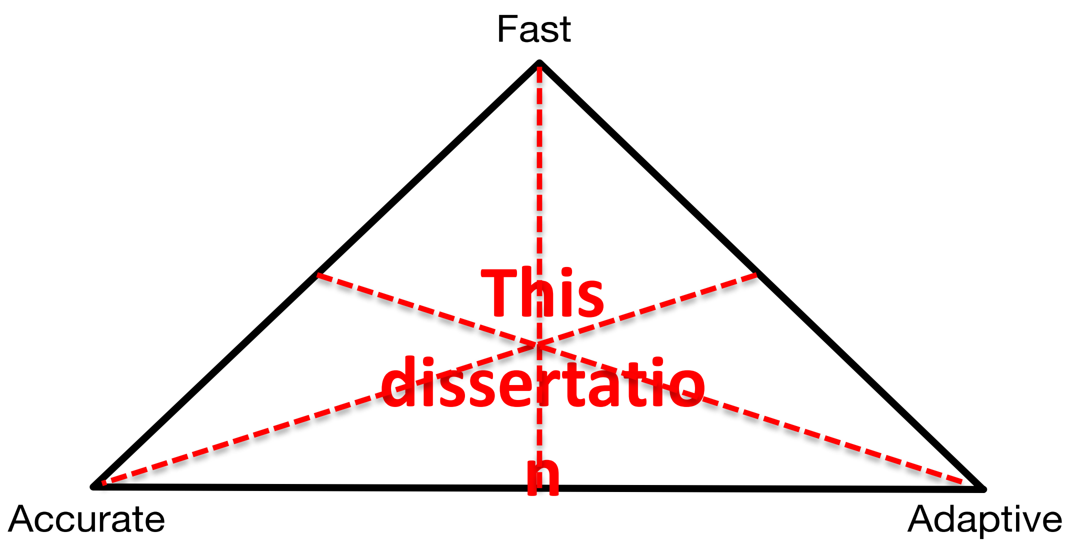
SUMMARY

- We propose a **generalized framework** to build **ECOC matrices** that follow a certain **error-correcting criterion design**.
- The **Error-Correcting Factorization** is formulated a **constrained Coordinate Descent**.
- We **allocate the correction capability** of the ECOC to those categories which are more **prone to confusion**.
- Experiments show that we obtain **higher accuracies** than state of the art methods **with more efficient models**.



Conclusions

- In this dissertation we have proposed approaches for optimizing ECOC classifiers based on various optimization methods.
    1. **Accurate**: by using powerful binary classifiers.
    2. **Adaptive**: exploiting multi-class data distribution.
    3. **Fast**: minimizing the number of classifiers used.



- We have proposed a **novel representation** of **Error-Correction** for an ECOC, enabling us to **allocate Error-Correction** in a flexible manner.

- We have tackled open questions regarding **Error-Correction capabilities** of ECOCs.

  - How do sub-linear ECOC matrices behave? ☑

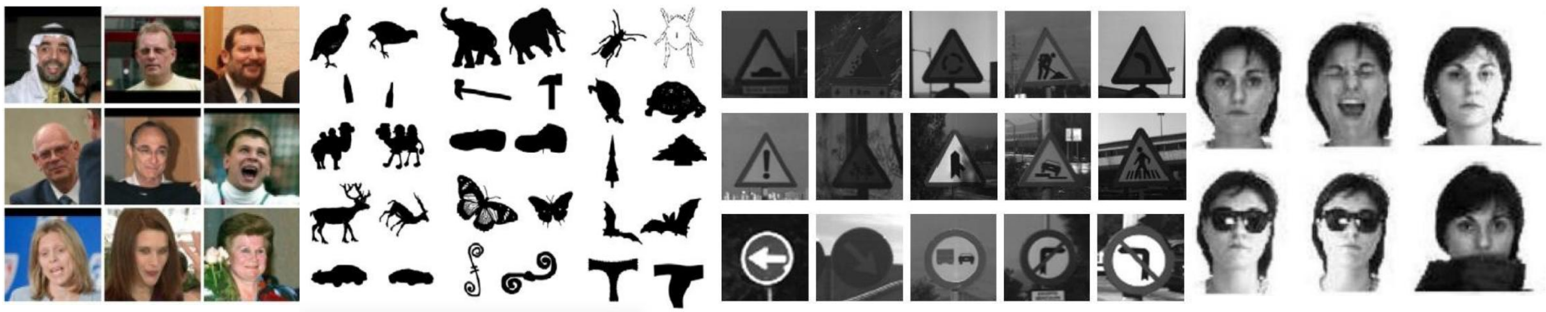
  - Is Error-Correcting distributed evenly on all classes? ☑

  - Can problem-dependent designs profit from the distribution of Error-Correcting capabilities? ☑

  - Is it better to allocate Error-Correction to classes prone to error or to classes not prone to error? ☑

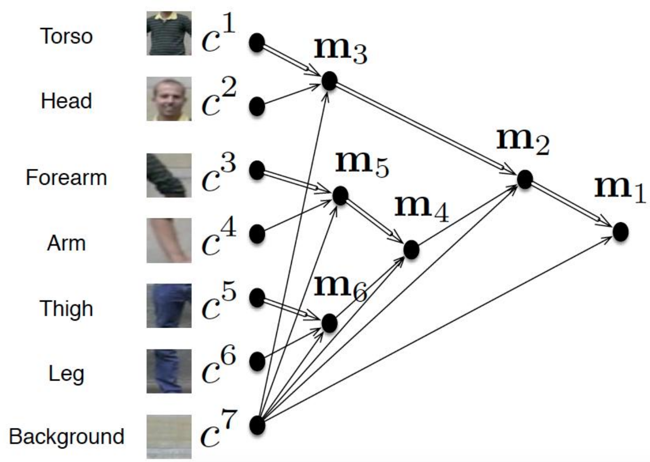
  - What is the minimum problem-dependent number of classifiers needed for an ECOC matrix? ☑

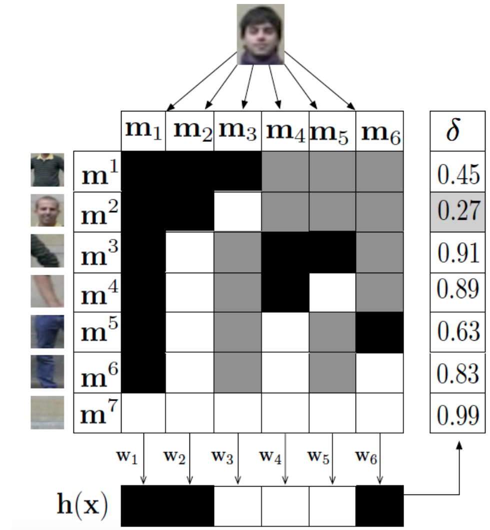
- We have **evaluated** the proposed approaches in **several Multi-class classification tasks:**
  - UCI datasets: Localization sites of proteins, Japanese vowel sounds, written letters, etc.
  - Computer Vision datasets: Face Recognition, Traffic sign recognition, symbol recognition, etc.



- Our approaches have **outperformed state-of-the-art** when analyzing accuracy as a function of the complexity.

- The proposed approaches have been evaluated in the challenging problem of **Human Pose Estimation**
  - Recognize **body limbs** of persons in an image.





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- **On the design of an ECOC-compliant genetic algorithm**. Bautista, Miguel Ángel and Escalera, Sergio and Baró, Xavier and Pujol, Oriol. **Pattern Recognition**, 47, 2, 865–884, 2013, Elsevier.

- **HuPBA 8k+: Dataset and ECOC-GraphCut based Segmentation of Human Limbs**. Sanchez, Daniel and Bautista, Miguel Ángel and Escalera, and Sergio. **Neurocomputing**, 2014, Elsevier.

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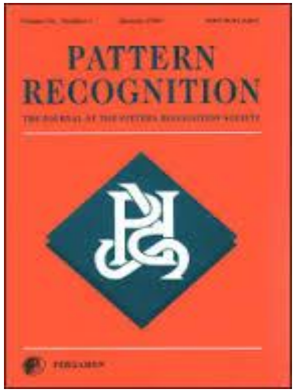
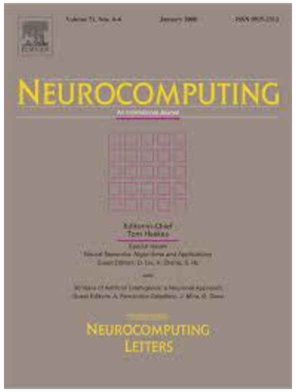
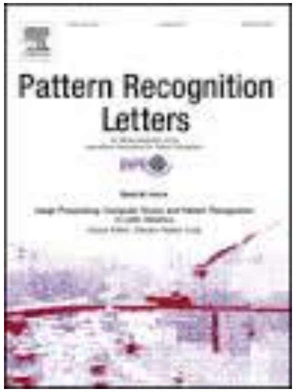
- **Probability-based dynamic time warping and bag-of-visual-and-depth-words for human gesture recognition in rgb-d**. Hernández-Vela, Antonio and Bautista, Miguel Ángel and Perez-Sala, Xavier and Ponce-López, Víctor and Escalera, Sergio and Baró, Xavier and Pujol, Oriol and Angulo, Cecilio. **Pattern Recognition Letters**, 50, 112-121, 2014, Elsevier.







- **Introducing the separability matrix for error correcting output codes coding**. Bautista, Miguel and Pujol, Oriol and Baró, Xavier and Escalera, Sergio. Multiple Classifier Systems, 227–236, 2011, Springer.

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- **Learning To Segment Humans By Stacking Their Body Parts**. Puertas, Eloi and Bautista, MA and Sanchez, Daniel and Escalera, Sergio and Pujol, Oriol. IEEE European Conference Computer Vision Looking At People Workshop, 2014.

Learning Error-Correcting Representations For Multi-class Problems

Miguel Ángel Bautista Martín

Thank you